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A NEW FORM OF CATHODE DISCHARGE AND THE PRODUCTION OF *X*-RAYS, TOGETHER WITH SOME NOTES ON DIFFRACTION.

PRELIMINARY COMMUNICATION.

By R. W. Wood.

IF accurate measurements are to be made of diffraction bands produced by the *x*-rays, it seems probable that very narrow slits, and a very intense and minute radiating source must be used. The focus tube has the disadvantage that the radiation is given off from a surface of considerable size, and is not very intense *per unit of area*, which is what is required for diffraction work. What we need is something that bears the same relation to the focus tube that the arc-light bears to a large flame. After considerable experimenting I have succeeded in finding a method of producing the rays by what appears to be a new form of cathode discharge, which manifests itself as a bright blue arc between two minute balls of platinum in a very high vacuum.

The advantage of the method is that a very small bulb can be used, an inch or less in diameter (of interest to those bent on getting a tube into the human body and lighting it up from within), which yields a radiation intense enough to show the bones of the forearm distinctly, from a source about the size of a small pin-head. *Per unit of area* of the radiating surface, I find the radiation to be between ten and twenty times as intense as with the

best focus tubes. This statement I base on photographs made of the two sources by means of a pin-hole 0.2 mm. in diameter.

I have not yet succeeded in producing a "total radiation" as intense as the large focus tubes yield, where the source is a plate of considerable area, but the method seems more rational than that of focusing the cathode rays on a platinum target, and I think it not unlikely that the development of a very powerful tube is possible on the lines indicated in this paper.

The apparatus is shown $\frac{1}{2}$ natural size in Fig. 1. The small bulb is blown on the top of a tube 80 cm. long, which dips into a jar of mercury. Within this tube is another, bent into a U at the bottom, one arm projecting out of the mercury. A conducting wire runs through this, and is attached to a platinum wire sealed through the glass at the top.

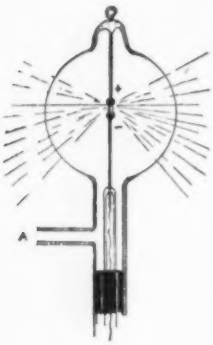


Fig. 1.

The arrangement is the same as that devised by the writer for determining the temperature variations in the stratified discharge, by means of a sliding bolometer, and will be better understood by referring to Vol. IV, No. 3, of the PHYSICAL REVIEW (Fig. 4). This rather complicated arrangement is not necessary at all, but it facilitates adjustments within the tube, and has been used thus far.

The electrodes are two platinum wires carrying small platinum balls, about 1.5 mm. in diameter, easily made by fusing the wire in the electric arc. One is fixed and the other can be moved up and down by means of the inner tube which supports it. The bulb is exhausted through the side tube *A*, which communicates with a mercury pump.

Starting with the balls about 5 mm. apart, the tube is exhausted (first clamping the projecting arm of the U) until, on passing a discharge through it, the green luminescence due to the cathode rays is very pronounced. The balls can be pushed within a millimeter of each other without producing any apparent change, the cathode rays being given off from the entire surface of the wire and ball.

If now a large spark gap be introduced into the external circuit, the green luminescence disappears as does also the bluish negative light, and the discharge passes between the balls in the form of a minute and brilliant blue arc. I use the word "arc" because on close inspection the discharge resembles an arc rather than a spark. It is intermittent of course. From the surface of the anode ball on which this arc plays, the *x*-rays emanate with great intensity.

I have been using a large 12-plate Wimshurst machine, which seems to work better than a coil. A good deal depends on the conditions: the platinum balls must be adjusted to the maximum striking distance (usually about 1 mm.). If they are too close together, the radiation is feeble. The external spark gap must also be adjusted carefully: if it is too short or too long, the arc disappears, we get the usual diffuse cathode rays, and the *x*-radiation vanishes except for the feeble effect from the glass.

When everything is just right, the little arc burns away steadily without interruptions, except the rapid periodic ones, and the anode radiates precisely as the positive carbon in the arc-light.

Another rather curious analogy between the discharge and the carbon arc is that the anode ball is eaten away to a shallow crater, while on the surface of the cathode is built up a dense, hard deposit of very brilliant metallic platinum. The appearance of the balls under the microscope, after a five-hour passage of the arc, is shown in Fig. 2. They are not visibly heated by the discharge, though at times showers of red-hot sparks are given off, which bounce about inside the bulb, and pour down into the barometer tube. The surface of the anode, where it has been eaten away, is pitted and corrugated but very brilliant: the built-up mass on the cathode has a similar appearance, but the rest of the surface is dull and without luster.

Fig. 2.

Another curious action of the arc is, that if it be made to play on the edge of a piece of platinum foil (used as anode), the edge is sharply crimped or turned up towards the cathode. In one tube in which the radiating anode was a small horizontal plate of platinum 0.5 cm. square, on the center of which the arc played, the

plate was bent into an arc of perhaps 60° concave towards the cathode. Although there is a strong attractive force between the anode and cathode (shown by the flying together of the balls if they are side by side), it does not seem to be sufficient to explain this action: moreover, the plate is not heated even to redness. The effect is such as would be produced if the surface on which the arc played was put in a state of tension, but as this surface is the one that is continually cut away by the discharge, it is difficult to imagine how such a state could be produced. A similar bending or crimping of the anode sometimes takes place in focus tubes.

I have already alluded to the spark showers which sometimes take place within the tube. On one occasion after running the current in one direction for several hours, I reversed it suddenly and observed a phenomenon which I have never seen surpassed in point of beauty in any of the various forms of vacuum tubes. From the anode there shot out two intense yellow beams resembling those of search lights shining through a mist (Fig. 3). The edges of the beams were as sharply defined as I have indicated in the drawing, and the closest scrutiny showed them structureless. Where they struck the wall of the bulb they apparently splashed off, and were somewhat brighter than at points midway between the source and the wall. The bulb was about 10 cm. in diameter, and the electrodes were shaped like those in Fig. 8, which will be alluded to later. The appearance of the beams was unlike anything that I have ever seen in a vacuum tube, and the only explanation that I could think of was that they might be caused by minute particles of heated platinum thrown off from the anode, too small to be perceived individually by the eye, but collectively presenting the appearance of a ray of light in smoky air. Subsequent events showed that beyond a doubt this was the

Fig. 3.

true explanation, for in a few minutes a close examination revealed the presence of bright lines in the rays which became more and more distinct, until no doubt remained that the yellow beams were in reality blasts of red-hot metallic particles which, at the outset, may have been of nearly molecular dimensions.

The phenomenon is undoubtedly caused by the rapid cutting away of the built-up deposit on the electrode to which I have already alluded, as I have observed it only when that electrode which has served for some time as cathode is suddenly made anode. The deposit built up by the arc, though apparently as hard and solid as the rest of the ball, is probably a dense aggregation of very minute particles, which is rapidly undermined and torn to pieces when the current is reversed. As we get deeper down into the mass, the projected particles become larger, until their individual trajectories can be seen; and the luminous beams become transformed into the common spark shower.

The behavior of these beams in a powerful magnetic field may be worth investigating, though it is doubtful if any effect would be found.

To return now to the tube shown in Fig. 1. By means of the pin-hole I photographed the radiating surface on a plate wrapped up in several thicknesses of black paper and obtained an image which is shown slightly magnified in Fig. 4. It will be seen that the under surface of the ball gives out most of the rays, while a few come from the wire: this, together with the fact that the extreme top of the bulb shines with a green light, leads me to think that the little arc is a very dense bundle of cathode rays, a few of which get around the ball and impinge, some on the wire and some on the glass. I have also noticed that particles of aluminum oxide on the upper surface of the anode ball, where no rectilinear rays from the cathode could possibly reach them, glow with a rich red color, while the arc plays between the balls, a fact which can only be explained by supposing that the cathode rays curve around and envelop the + electrode in the vicinity of the arc. If the balls are pushed nearer together, an aureole a centimeter or more in diameter surrounds the arc, which is bluish white with

Fig. 4.

platinum electrodes and bright green with copper. There are a number of points about the arc that require investigation. I find that it can be produced to a less degree between the cathode and a metal plate insulated on a glass stem, the anode being below. This suggests that oscillations may be set up, and it may be possible to construct a vibrator for short waves in some such way. The spectrum of the arc and its behavior in the magnetic field are also worth investigating. Most of the light comes from a film on the anode, which is sometimes yellowish, though usually blue. A more complete study of the nature of the discharge and the distribution of the cathode rays about it will be made in the near future.

Reference to Fig. 4 shows us (considering that the ball is but 1.5 mm. in diameter) that the chief source of the rays is an exceedingly narrow horizontal line, especially if the ball has been somewhat cut away, and we might therefore expect some diffraction phenomena to show themselves on the edges of shadows of objects thrown by such a source. This I find to be apparently the case. If a piece of fine copper wire 0.2 mm. in diameter be fastened horizontally on the outside of the bulb opposite the balls, and another wire vertically, making a +, the horizontal wire, which is parallel to the narrow radiating source, throws a sharp shadow on a plate 40 cm. away, while the vertical wire throws scarcely any shadow at all.

The shadow of the wire, which is of course light on the photographic plate, has a dark border, darker than the rest of the surface which receives the whole radiation, and there seems no other way of explaining this increased blackening on the edge of the shadow than by interference. Shadows of a wire mounted within the tube also showed this dark border. The distance of the wire from the radiating source was about 1.5 cm., and the plate was about 40 cm. from the wire, so that the shadow of the wire was nearly half a centimeter wide, a magnification of about 20 diameters: in spite of this it was in most cases sharply defined.

The dark border was only to be found *on one edge;* namely, the one towards the radiating anode. The other edge appeared slightly lighter than the rest of the shadow, somewhat as indicated

in Fig. 5. If the upper ball was the anode, the dark border was on the upper side of the shadow; if, however, the lower ball was

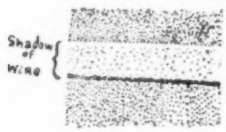


Fig. 5.

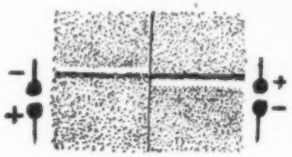


Fig. 6.

made anode by reversing the current, the light and shade were reversed. (Fig. 6: both exposures made on the same plate, one half of it being screened each time with a strip of lead.)

I see no way of explaining these effects except by diffraction or something analogous to it, for no half-shadows or penumbræ will account for the edge of the wire's shadow being darker than the rest of the plate which is exposed to the full action of the rays. There seemed at first sight to be no way of explaining the difference in the appearance of the two edges of the shadow; or in other words, why the edge of the wire towards the radiating surface should act differently from the one facing away from it. Some rotational element in the rays, such as would exist if they were vortex filaments in the plane of the wire, might cause them to be affected differently by the two edges, but such an explanation is hardly worth considering, at least on present evidence.

By combining diffraction with penumbral effects we might possibly get a similar result: reference to Fig. 4 shows that the radiation, though most intense from the under side of the ball, comes also to a less degree from the sides as the arc plays about over the surface.

Draw a figure showing the paths of two pencils of rays; a strong one from the under surface and a weaker one from a point above, and suppose the two edges of the wire's shadow thrown by the first pencil to have dark edges due to interference maxima. It will be seen that the feebler rays coming from the upper source will cast a shadow of the wire a trifle lower down than the first shadow, but will illuminate all the rest of the plate. If this shadow falls on the under dark band, it will tend to obliterate it, if the

relative intensities are about equal; that is, if the darkening due to the rays from the upper source is about equal to the increased darkening due to interference. If the lower ball is made anode, the conditions are reversed, the feebler source being below the brighter, and we have the maximum on the upper edge of the shadow obliterated.

To remove all chances of penumbræ a much narrower radiating source is necessary, and I made a great number of experiments to determine the best possible form to give the apparatus. I first tried a long narrow bulb with the platinum beads very close to the glass, and a slit .05 mm. wide fastened with wax to the outside just opposite the anode. This form, though giving excellent results at first, was very short lived and very apt to be punctured by the discharge. Better results were obtained with a form shown in Fig. 7. Over the mouth of a thistle tube was fastened with sealing wax a sheet of very thin aluminum plate, which served as cathode. The anode was a platinum bead which is pushed up until the arc plays between the plate and the ball; the *x*-rays coming from the ball pass through the aluminum plate, on which is fastened a narrow slit of copper or platinum. The chief advantage of this form is that the cutting away of the anode does not alter its position with reference to the slit, though the deposit of platinum on the aluminum plate soon lessens its transparency.

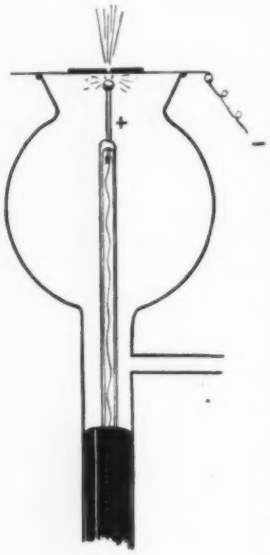


Fig. 7.

The best form found thus far is shown in Fig. 8. A rectangular plate of platinum foil is partially cut across with the point of a very sharp penknife: this makes a slit about 0.05 mm. wide with very uniform straight edges. The plate is soldered with gold to the wire carrying the cathode, which is a short piece of thick plati-

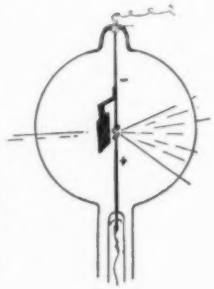


Fig. 8.

num wire mounted parallel to the slit. The anode is of similar form and is mounted on the movable rod, and the arc plays back and forth between the two cylindrical surfaces.

The diffraction slit, furnished with a micrometer screw, is placed at a distance of about 15 cm., as nearly as possible in the center of the beam which comes through the knife cut in the platinum plate (the proper position being found by means of a small screen of calcium tungstate), and the photographic plate behind this, at a distance of 25 cm. It is important that the electrode carrying the plate be made cathode, otherwise *x*-rays are given off in profusion from both its surfaces as shown by a pin-hole photograph.

I tried one other arrangement which promises to be even better than this, in that it furnishes radiations from a source 0.05 mm. in diameter without the use of a slit, which makes the position of the diffraction slit immaterial, providing it be only parallel to the source. This method is to allow the cathode arc to play upon a horizontal and tightly stretched bit of platinum wire 0.05 mm. in diameter, and gives beautiful definition while it lasts, but unfortunately the discharge cuts through the wire in ten or fifteen minutes.

I have designed a tube in which new wire can be continually fed to the arc, which I think may be better adapted to diffraction work than the one with the slit.

The images of the slits on the photographic plate give evidence of a maximum darkening on each edge. With a comparatively wide slit (0.2 mm.) the appearance is about as indicated in Fig. 9 (magnified). This was obtained with the plate at a distance of about 15 cm. from the slit. With a slit 0.05 mm. wide at a distance of 25 cm. the image was uniformly dark, but there was a very faint trace of a light border followed by a faint shadow. This looked more like true diffraction than any of the other results, and for comparison a plate was made under similar conditions with light. A rough calculation based on these two plates would indicate a wave length of about 0.00004 for the *x*-rays, a trifle less than one half that of Schumann's shortest light wave. I place very little confidence in this estimate, and in fact am not yet at all

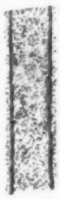


Fig. 9.

sure that we are dealing with true diffraction. There seems to be very strong evidence, however, that the rays do not pass by the edges of obstacles in rectilinear paths without suffering any change, although there remains a good deal of work to be done before any very definite statements as to just what happens can be made.

It does not seem to me to be unreasonable to suppose that there may be some action which is not identical with the interference of light and heat waves.

In conclusion, I wish to express my thanks to Professor Charles R. Cross for his great kindness in placing at my disposal the facilities of the Rogers Laboratory of Physics of the Massachusetts Institute of Technology.

Jamaica Plain, March 27, 1897.

AN EXPERIMENTAL STUDY OF INDUCTION PHENOMENA IN ALTERNATING CURRENT CIRCUITS.

By F. E. Millis.

2. *Resonance in Alternating Current Circuits.*

(Concluded from Vol. IV., Page 142.)

IN a circuit containing an electromotive force, resistance, self-induction, and capacity, the equation by which the current may be computed from the electromotive force is

$$e = Ri + L\frac{di}{dt} + \frac{q}{c}, \tag{8}$$

in which e is the impressed electromotive force; R, the resistance; L, the self-induction; C, the capacity; i, the current at any instant of time t; and q, the charge of the circuit at any instant.

Ri represents the electromotive force necessary to overcome the ohmic resistance; $L\frac{di}{dt}$, that necessary to overcome the counter-electromotive force of self-induction; and $\frac{q}{c}$, that necessary to overcome the counter-electromotive force due to the capacity of the circuit; and their sum equals the impressed electromotive force e. If the impressed electromotive force is periodic, and a single-valued function of the time, it may be represented by $e = f(t)$.

This may be written by Fourier's theorem:

$$e = A_0 + \sum_{n=1}^{n=\infty}(A_n \sin n\omega t + B_n \cos n\omega t), \tag{9}$$

where n takes all plus integer values; $\omega = 2\frac{\pi}{\tau}$, τ being the time of one period; t the time at any instant counted from the beginning of the wave, and A_1, A_2, etc., B_1, B_2, etc., are constants to be determined. A_0 is the average algebraic sum of the ordinates, and will equal zero if the line which divides the curves so that the

areas of the negative waves shall be equal to the areas of the positive waves, be taken as the zero line from which to measure the ordinates.

Any two waves having the same period, and of the forms $y_1 = a \sin \omega t$ and $y_2 = b \cos \omega t$, may be compounded into the single sine wave $Y = K \sin (\omega t + \phi)$, where $K = \sqrt{A^2 + B^2}$, and $\tan \phi = \frac{B}{A}$, since the components are at right angles. Hence equation (9) may be written:

$$e = \sum_{n=1}^{n=\infty} K_n \sin (n\omega t + \phi_n) = f(t). \qquad (10)$$

With this expression for the electromotive force, we obtain from (8) the following expression for the current:

$$i = \sum_{n=1}^{n=\infty} \frac{Kn}{\sqrt{R^2 + \left(\frac{1}{n\omega C} - n\omega L\right)^2}} \sin \left[n\omega t + \phi_n + \tan^{-1}\left(\frac{1}{n\omega CR} - \frac{n\omega L}{R}\right)\right] + k_1 \epsilon^{-\frac{t}{H_1}} + k_2 \epsilon^{-\frac{t}{H_2}}, \qquad (11)$$

which is a general expression for the periodic current flowing in a circuit, there being as many terms in the summation as there are in the impressed electromotive force.

Here k_1 and k_2 are integration constants, ϵ is the Naperian base, $H_1 = \frac{2LC}{RC - \sqrt{R^2C^2 - CL}}$, and $H_2 = \frac{2LC}{RC + \sqrt{R^2C^2 - CL}}$.

The positive and negative waves of electromotive force are usually symmetrical, hence only odd harmonics will be present. Usually only the third or fifth harmonic will be appreciable; hence

$$e = A_0 + A_1 \sin \omega t + B_1 \cos \omega t + A_3 \sin 3\omega t + B_3 \cos 3\omega t + A_5 \sin 5\omega t + B_5 \cos 5\omega t \qquad (12)$$

may be taken as the complete expression for the impressed electromotive force. This equation may be written by (10).

$$e = A_0 + K_1 \sin (\omega t + \phi_1) + K_3 \sin (3\omega t + \phi_3) + K_5 \sin (5\omega t + \phi_5), \qquad (13)$$

in which

$$K_1 = \sqrt{A_1^2 + B_1^2},\ K_3 = \sqrt{A_3^2 + B_3^2},\ K_5 = \sqrt{A_5^2 + B_5^2},$$

$$\tan \phi_1 = \frac{B_1}{A_1},\ \tan \phi_3 = \frac{B_3}{A_3},\ \text{and}\ \tan \phi_5 = \frac{B_5}{A_5}.$$

If one term of this electromotive force has the same period as the circuit to which it is applied, the amplitude of this harmonic will be greatly increased in the resulting curve. The natural period of the circuit will depend upon the capacity and self-induction in the circuit, the capacity and self-induction being so chosen that the charge and discharge of the condenser will be oscillatory. The period of this oscillation may be made the same as that of the particular harmonic in the impressed electromotive force.

The form of the electromotive force impressed by a particular dynamo can be determined by obtaining the curve of electromotive force and analyzing it. This analysis gives the constants and the phase relations of the various components of equations (12) and (13), and by it is determined the number of harmonics present in the impressed electromotive force and their respective amplitudes. The principles on which this analysis is based are given in Thompson and Tait's *Natural Philosophy*, Vol. I., p. 54, and in Donkin's *Acoustics*, p. 56, and following.

Briefly the analysis is as follows: Measure a large number of equally spaced ordinates in one complete period of the curve, and then multiply each ordinate by the sine of the angle which corresponds to the position of the ordinate in the curve. Divide the algebraic sum of all the products by the number of ordinates measured, and twice this quotient is A_1, the amplitude of the sine component of the fundamental curve. The ordinates multiplied by the cosines of the corresponding angles, and treated in the same way, give the amplitude of the cosine component of the fundamental. The amplitude of the harmonic components are determined in the same manner by multiplying the ordinates by the corresponding sines and cosines of the harmonic, one degree in the fundamental wave corresponding to three degrees in the third harmonic, and five degrees in the fifth harmonic.

The above analysis applies to periodic curves, but the current is of interest before it reaches its fixed form after the circuit has been closed.

In equation (11) the exponential terms decrease as t increases, and become inappreciable in a fraction of a second; but they modify the character of the curve considerably when the circuit is first closed.

The exponential term may be written [1]

$$i_1 = A\epsilon^{-\frac{Rt}{2L}} \sin\left\{\frac{\sqrt{4LC - R^2C^2}}{2LC}t + \phi\right\}, \qquad (14)$$

where A and ϕ are constants of integration.

These constants are determined by the condition that $i = 0$ and $q = 0$, when the circuit is closed. They are found to be

$$\phi = \cot^{-1} - \left\{\frac{2\cot\psi_1 + RC\omega}{\omega\sqrt{4CL - R^2C^2}}\right\} - \frac{\sqrt{4CL - R^2C^2}}{2CL}t_1, \qquad (15)$$

and

$$A = -\frac{2I\epsilon^{+\frac{Rt_1}{2L}}}{\omega\sqrt{4CL - R^2C^2}}\sqrt{(CL\omega^2 - 1)\sin^2\psi_1 + \tfrac{1}{2}RC\omega\sin 2\psi_1 + 1}, \qquad (16)$$

in which I equals the coefficient of the sine in (11), ψ equals the angle of which the sine is taken in (11), and ψ_1 equals the value of ψ at time t_1, when the circuit is closed.

Substituting these values of A and ϕ in (14), the general equation (11) may be written

$$i = \Sigma I\sin\psi - \frac{2I\sqrt{(LC\omega^2 - 1)\sin^2\psi_1 + \frac{1}{2}RC\omega\sin 2\psi_1 + 1}}{\omega\sqrt{4LC - R^2C^2}}\epsilon^{-\frac{R}{2L}(t - t_1)}$$
$$\sin\left\{\frac{\sqrt{4LC - R^2C^2}}{2LC}(t - t_1) + \cot^{-1} - \left[\frac{2\cot\psi_1 + CR\omega}{\omega\sqrt{4LC - R^2C^2}}\right]\right\}. \qquad (17)$$

This equation means that i will be made up of two oscillating curves. The curve represented by the first term is a uniform curve of a permanent type, while the one represented by the second term dies down rapidly after the electromotive force is first introduced, the rate of dying away being determined by the value of $\frac{2L}{R}$ in the exponent of ϵ. The initial value of this logarithmic decrement curve is given by the coefficient of ϵ. This value will depend upon ψ_1.

When $t = t_1$, the first term is $\sin\psi_1$, and the second term becomes $-I\sin\psi_1$. Hence, upon closing the circuit the current starts from zero, and the direction and magnitude of the first wave depends upon the relative phases and magnitudes of its two components.

[1] Alternating Currents, Bedell and Crehore, p. 95.

By the *phase* in which a circuit is closed is meant the phase that the current would have at that instant if it *started* as an harmonic current of its permanent type, having the same phase difference with the electromotive force as it finally assumes.

The phase at which the short-lived current begins is determined by its angle of lag relative to the permanent component. The cotangent of this angle is

$$-\left(\frac{2\cot\psi_1 + CR\omega}{\omega\sqrt{4CL - R^2C^2}}\right);$$

the phase ψ of the permanent current in which the circuit is closed, may be found by beginning with the curve after the short-lived component has become inappreciable, and plotting this permanent curve back to the point where the circuit was closed, or what amounts to the same thing, by superposing upon the irregular curve a curve of the permanent type.

Curve No. 12 is the curve of a transformer when fed by a Brush alternator. The curve was obtained with the instrument described in this Journal, Vol. IV, p. 132, and though primarily a current curve, it is similar to the electromotive force curve, since the circuit contained neither self-induction nor capacity. Analysis shows it to be represented by the following equation:

$$e = 1.7026\sin\omega t - 0.00013\cos\omega t - 0.0185\sin 3\omega t - 0.0156\cos 3\omega t.$$

The fundamental makes 117 alternations per second, as shown by the tuning-fork trace on the plate. Hence its third harmonic makes 351 alternations per second.

Curves Nos. 13, 14, and 15 were obtained from the same transformer and alternator as No. 12, but the circuit contained condensers and a self-induction coil in series, the number of condensers in circuit being different for the different curves. The conditions were otherwise unchanged, except that the alternator was not running at quite the same speed in all cases.

Curve No. 16 was obtained from the same transformer and under the same conditions as was No. 12, except the transformer was supplied by a Thomson-Houston alternator, curve No. 17 was from the transformer and Thomson-Houston alternator with con-

densers and self-induction coil in the circuit, the self-induction and capacity being the same as in No. 13.

Curves No. 18 and 19 were obtained with the same constants in the circuit as was No. 13, but show the effect of the exponential terms in equation (14). The three curves are identical after these terms have become negligible.

Nos. 18 and 19 differ only in that the falling plate happened to close the circuit in different phases.

The phase of closing the circuit in No. 18 was determined by superposition, as indicated above. The phase of the lag of the short-lived curve, the initial value of its logarithmic decrement, the time constant of this decrement curve, and the maximum value, I, of the permanent curve were then computed, and with these values curves were plotted which agree very closely with the curve obtained from the machine.

The accompanying table gives the constants for the different curves.

In this table it is assumed that the self-induction of the transformer is negligible in this circuit, so that the period of the condenser, the time constant of the circuit, and other expressions involving self-induction are not changed by the influence of the transformer.

This assumption is allowable because the alternator and transformer serve to produce a potential difference at the transformer terminals, varying according to a certain law, and this law of variation will not be changed by connecting the terminals through a circuit except in so far as the resulting current reacts upon the transformer. In the present case a large transformer was used, and the current through the secondary circuit was never greater than 0.26 ampere, surely too small to change either the form or the magnitude of the electromotive force. An examination of the table shows that the presence of the condensers emphasizes the third harmonic in every case, but a comparison of curves 12, 13, 14, and 15 is especially interesting, since they are all obtained from the same type of electromotive force.

The values of the constants show that in curve 12, when the condensers are not in circuit, the amplitude of the fundamental is

Constants.	Curve No. 12. Transformer supplied by Brush alternator.	Curve No. 13. Obtained from No. 12.	Curve No. 14. Obtained from No. 12.	Curve No. 15. Obtained from No. 12.	Curve No. 16. Transformer supplied by T. H alternator.	Curve No. 17. Obtained from No. 16.	Curve No. 18. Obtained from No. 12.	Curve No. 19. Obtained from No. 12.
R, ohms	360.000	5.100	5.100	5.100	360.000	5.100	5.100	5.100
L, henrys . . .	0.000	0.036	0.036	0.036	0.000	0.036	0.036	0.036
C, microfarads . .	0.000	8.310	6.780	5.540	0.000	8.310	8.310	8.310
Periodicity $= \frac{1}{T_a}$, of alternator . . .	117.000	116.500	111.000	113.000	125.000	125.000 ?	112.000	112.000
Periodicity $= \frac{1}{T_b}$, of 3d harmonic . .	351.000	349.500	333.000	339.000	375.000	375.000 ?	336.000	336.000
Periodicity $= \frac{1}{T_c}$, of condenser . . .		300.000	328.000	360.000		300.000	300.000	300.000
K_1	1.702	2.880	2.090	1.670	1.703	3.040		
K_3	0.027	0.710	1.860	0.830	0.227	2.080		
K_5					0.215	0.530	Same as No. 13 after they	
ϕ_1	− 0° 0′ 18″	−2° 10′	−19° 55′	−36° 30′	−2° 10′	−10′ 25″	have become permanent.	
ϕ_3	+40° 10′	+1° 50′	+ 9° 40′	+23° 20′	−35° 00′	+ 6° 43′		
ϕ_5					+2° 10′	−38° 40′		

sixty-three times that of the third harmonic. In No. 13, though the period of the condensers lacks considerable of being in unison with the third harmonic, yet the amplitude of the fundamental is only *four* times that of the third harmonic. In No. 15, with more nearly unison, the amplitudes are in the ratio of *two* to *one*, and in No. 14, with still better unison, the amplitude of the fundamental is only *one and twelve-hundredths* times that of the harmonic.

Nor is this increase in the amplitude of the *third* harmonic the only change produced by varying the capacity of the circuit. ϕ_3 is given in the table in terms of the angles for the *third* harmonics. Dividing by three to reduce it to degrees of the fundamental, and taking account of the algebraic signs of the sine and the cosine components of the harmonic, the position of the harmonic may be expressed by saying that it is 13° 22′ ahead of opposition in phase to the fundamental in curve No. 12, 3° 25′ ahead in No. 13, 23° 10′ ahead in No. 14, and 44° 20′ ahead in No. 15.

The close agreement of the curves obtained by plotting the fundamentals and *third* harmonics found by the analysis of these curves shows that no higher harmonics are present, but the *fifth* harmonic has almost as large an amplitude in No. 16 as has the third. Note also that the condensers placed in the circuit to obtain No. 17 increased both the fundamental and the fifth harmonic, but in not nearly so large a ratio as the *third* harmonic is increased, and that with the condensers as far out of unison as in any case tried. The increase of the fundamental is due to the fact that the resistance is decreased in greater effective ratio than the self-induction and capacity are increased rather than to any true resonance.

Referring to equations (11) and (17) for curve No. 18, I is computed to be 0.037 C. G. S. units, the initial value of the logarithmic decrement curve = 0.0578, $\psi_1 = 61° 24'$, $\chi = -34° 30'$, the time-constant = 0.014 sec., and the period of the short-lived current = 0.0034 sec. Plotting the first periods with these values gives close agreement with the original curve.

Lantern slides were made from the negatives with the camera. With these slides the curves were thrown on paper screens, enlarged to about six times the size of the cuts here given, and carefully traced with a pencil.

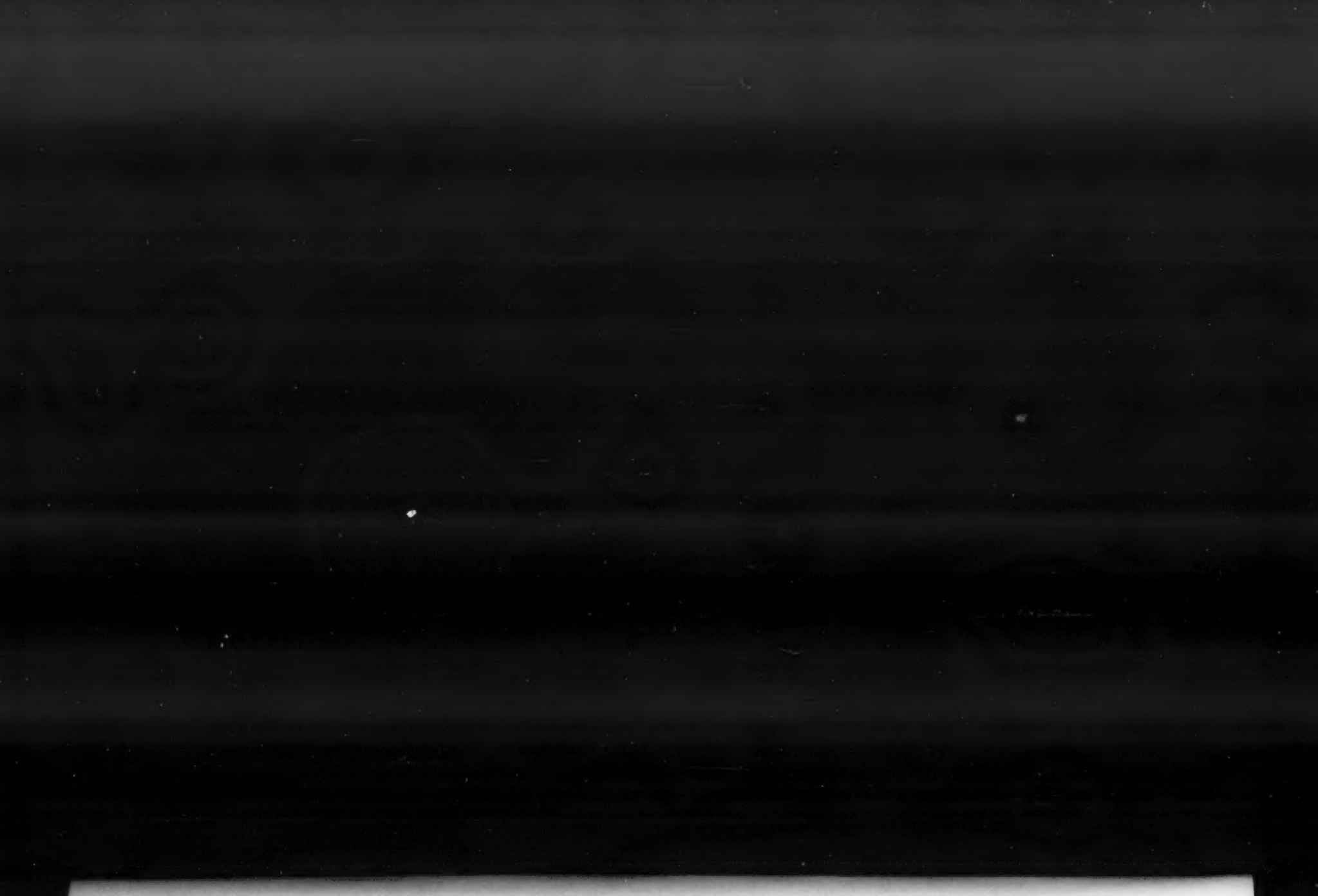
From these tracings the measurements were made. The measurements were expressed in inches, which gave the constants expressing the amplitudes of the components their particular value.

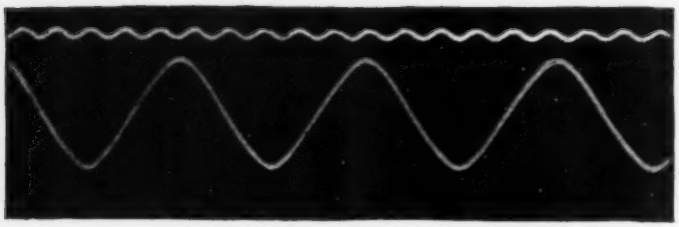
ORDINATES OF CURVES.

	No. 14.		No. 15.		No. 16.	
	Ordinates measured on original curve.	Ordinates measured on platted curve.	Ordinates measured on original curve.	Ordinates measured on platted curve.	Ordinates measured on original curve.	Ordinates measured on platted curve.
1	−1.30	−1.36	−1.40	−1.40	0.075	0.075
2	−1.56	−1.62	−1.38	−1.38	0.125	0.100
3	−1.75	−1.76	−1.29	−1.33	0.200	0.180
4	−1.81	−1.70	−1.16	−1.18	0.280	0.300
5	−1.66	−1.45	−0.96	−0.90	0.420	0.460
6	−1.20	−1.05	−0.73	−0.62	0.610	0.660
7	−0.72	−0.53	−0.34	−0.24	0.870	0.910
8	0.05	0.15	+0.10	+0.15	1.115	1.160
9	0.99	0.86	0.45	0.60	1.380	1.280
10	1.70	1.59	0.82	1.00	1.500	1.490
11	2.34	2.25	1.22	1.38	1.580	1.550
12	3.05	2.95	1.66	1.70	1.635	1.580
13	3.46	3.36	1.96	1.93	1.670	1.600
14	3.68	3.72	2.08	2.06	1.700	1.640
15	3.76	3.80	2.17	2.10	1.730	1.660
16	3.69	3.73	2.15	2.04	1.760	1.710
17	3.34	3.43	2.02	1.92	1.790	1.760
18	2.85	2.95	1.82	1.78	1.850	1.840
19	2.29	2.38	1.69	1.54	1.910	1.880
20	1.81	1.80	1.31	1.34	1.950	1.840
21	1.34	1.26	1.16	1.14	1.840	1.700
22	0.70	0.55	0.95	0.95	1.470	1.500
23	0.30	0.21	0.84	0.83	1.060	1.180
24	−0.02	−0.14	0.77	0.74	0.700	0.900
25	−0.20	−0.25	0.79	0.75	9.495	0.600
26	−0.17	−0.20	0.88	0.84	0.330	0.340
27	0.00	0.00	0.98	0.94	0.225	0.160
28	+0.27	+0.25	1.14	1.07	0.135	0.060
29	0.54	0.60	1.24	1.21	0.073	−0.030
30	1.03	0.95	1.31	1.30	0.000	−0.060

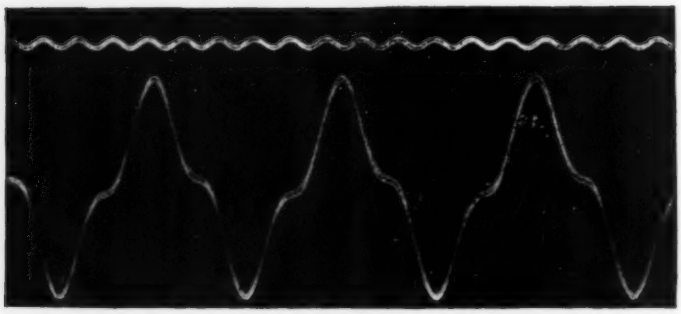
Ordinates were measured every six degrees; that is, sixty ordinates were measured in every complete period. A comparison of the ordinates measured on the curves with corresponding ordinates

obtained by plotting the component curves with their phase differences, as computed from the analysis, gives a much better idea of the degree of similarity between the curves than the mere placing of the plotted curve beside the original. That such a comparison may readily be made, a table is given containing side by side the ordinates of one-half period of each of three curves. The greatest discrepancy in any instance is 0.21 inch, and the two curves lie very close together, even for that difference, since at that portion of the curves they are falling rapidly.

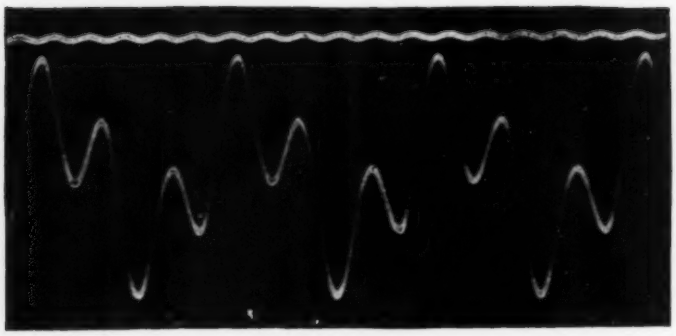




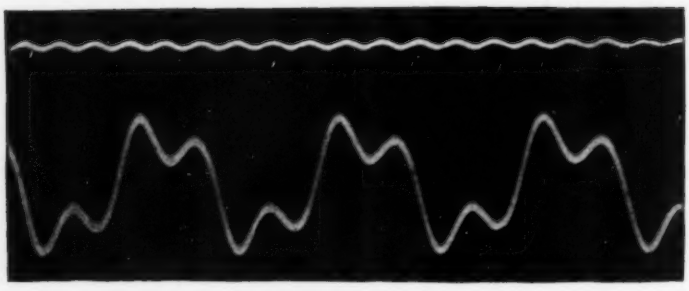
Curve 12.



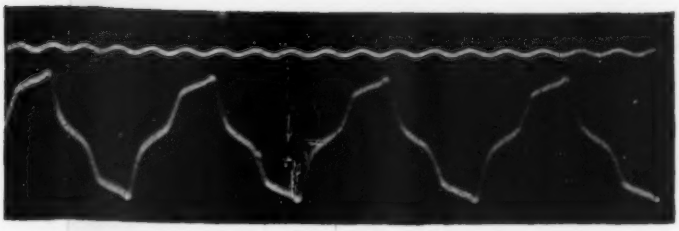
Curve 13.



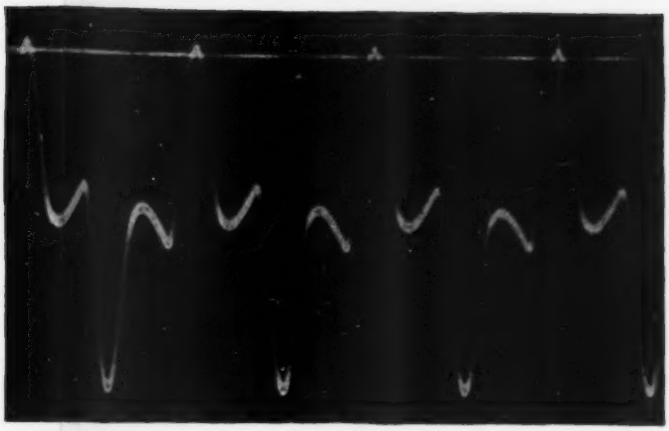
Curve 14.



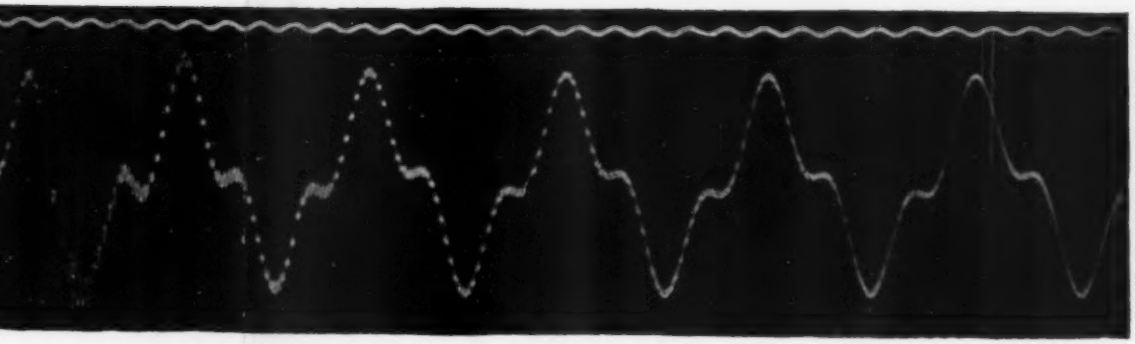
Curve 15.



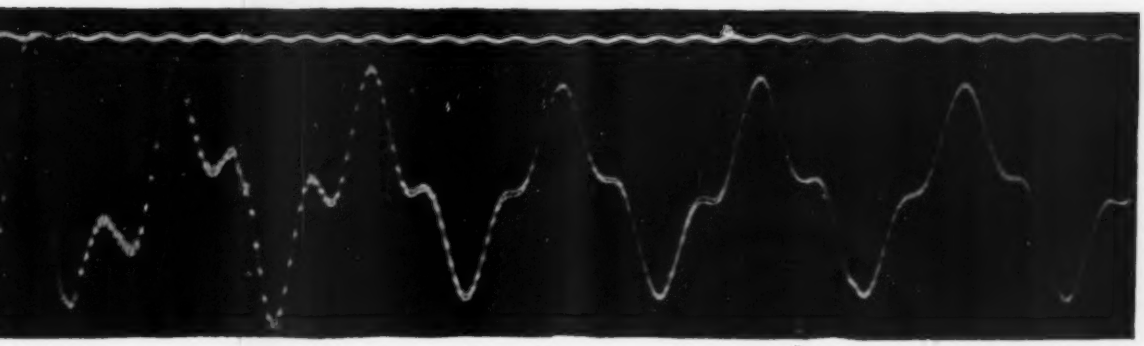
Curve 16.



Curve 17.



Curve 18.



Curve 19.

ON THE CONVERSION OF ELECTRIC ENERGY IN DIELECTRICS.[1] II.

By Richard Threlfall.

TWO ebonite spheres cut from the same rod of the Silvertown Rubber Company's best ebonite. The smaller sphere was formed from the central part of the rod, while the larger was only slightly less in diameter than the rod itself.

The dimensions are —

Diameter, larger sphere	1.5 cm.
Diameter, smaller sphere	0.713 cm.
Mass, larger sphere	2.097 g.
Mass, smaller sphere	0.214 g.

The resulting curves were the same whatever the surface of the ebonite. For the larger sphere,

$$W = 0.017\, F^{1.85},$$

and for the smaller,

$$W = 0.03\, F^{1.85}.$$

The index of F was absolutely the same for the two spheres. The great difference in the coefficients, however, excited surprise at the time, as no such inconsistency had been met with up to then, and the experiments were therefore repeated and varied in every possible way. For instance, the same quartz fiber was used for both spheres so as to get different deflections; then separate threads were used so as to make the deflections about equal. The spheres were dried and warmed by flame gases and tested one after the other over the whole range as quickly as possible. In another experiment each sphere was dried for several days, as usual, before measurements were made.

All these trials resulted in the same thing — identity of index, and variability of coefficient of F. The only possible explanations of the difference are, either that an outer shell of an ebonite rod differs from the core, or that the difference of size with respect to the guard tube has a specific effect. The latter hypothesis was tested, as far as possible, by casting spheres of different materials of different sizes, but it soon appeared that the spheres were so variable in property among themselves that no two, even of the same size, could be got to behave alike. Consequently, I can-

[1] Continued from the Physical Review, Vol. IV., p. 479.

not prove formally that the observed difference with the ebonite spheres is not due to the action of the guard tube, but, taken in conjunction with the other facts, there is practically no doubt as to the difference being real.

The relation between speed and deflection was tested with the smaller sphere, with the following results. Potential difference, 40,000 volts.

Frequency of rotation.	Double deflection in centimeters.
9.1	26.45
16.0	26.64
25.0	26.44

GLASS.

Only one kind of glass was used, viz. English flint glass made by Messrs. Powell & Co., of Whitefriars. Out of this two spheroids and several cylinders were constructed, and several trials were made with them. Owing to a variety of reasons, observations were practically limited to one spheroid.

Longer axis	2.244 cm.
Shorter axes	0.926 cm.
Density	3.1306
Volume	1.0107 (measured, as the figure was not very perfect).

With regard to the value to be assigned to the specific inductive capacity, the data was not very satisfactory. Hopkinson measured the specific inductive capacity of a sample of Chance's "Light Flint," which has a density of 3.2, and is therefore probably very similar to Powell's glass. The Light Flint gave a value for K which may be taken at 6.7.

This, with the above dimensions, leads to an internal screening factor of 0.233.

A large number of experiments were made with this spheroid. The relation between potential difference and angular deflection was examined in the three tubes — glass, mica, and ebonite — at twelve points between 0 and 6000 volts. The above experiments were repeated in two tubes, the glass being unvarnished, and were also performed both for varnished and unvarnished glass at high and at low frequency of field rotation. It was found that varnishing the glass, or varying the speed of rotation, or varying the guard tube, had absolutely no effect on the index of F, though, of course, the coefficient of F varied, according to the guard tube employed, owing to the screening inconsistencies. These experiments, more than any others, were relied upon in establishing the validity of the experimental method. A great number of measurements were also made with the spheroid and guard tube in different conditions as to surface moisture, and,

indeed, it was in this way that the dependence of the results upon the hygrometric state of the air was first discovered. The final result arrived at for varnished glass, when the tube and spheroid were both dry, — temperature 26° C. and frequency of rotation 18.8, — is

$$W = 0.038\, F^{1.92}.$$

When $F = 1$, the loss is about 14 per cent, and the percentage loss for other values of F is

$$\text{Percentage loss} = 14 \times F^{\,0.08}.$$

The curve was not very even, owing to the assistant at the voltmeter not having acquired sufficient skill, and, therefore, a special examination was made to find whether an index of 2 would not conceivably satisfy the results. This proved to be impossible.

For comparison, the following results are stated; they were obtained under varying conditions of dryness of the spheroid and tube, and illustrate the importance of having things dry. The index is correct but the coefficient approximate.

(1) Guard tube dry; glass dried carefully by hot gases, but not left over phosphorus pentoxide:

$$W = 0.026\, F^{1.85}. \quad \text{Frequency, 26.}$$

(2) Ditto. Frequency, 4 per second:

$$W = 0.036\, F^{1.85}.$$

(3) Spheroid dried by P_2O_5; tube damp. Frequency, 6:

$$W = 0.07\, F^{1.72}.$$

(4) Very damp day; spheroid dried by P_2O_5 for 4 days:

$$W = 0.03\, F^{2.2}. \quad \text{Frequency, 25.}$$

On this occasion the tube was perceptibly deficient in insulating power, and this is almost tne only experiment which showed a rise of index due to dampness. Since it is probable that the tube became electrified quite unevenly and that the field was far from uniform, no weight is to be attributed to the result, and it is only introduced here to show how great an effect dampness may have on the results. On blowing hot air on the guard tube the deflection at once increased.

Instead of tracing the variation of the coefficients with change of frequency, it will suffice to reproduce two sets of deflections at varying speeds. Of these, one refers to experiments on the spheroid on a dry day — when the tube was dry — and the spheroid dried by hot air before being mounted in the tube, but not by P_2O_5, and the other to the case of both the tube and spheroid being as dry as possible, *i.e.* to the same set of experiments as gave the value of W adopted here.

SPEED AND DEFLECTION.

Spheroid of varnished glass, imperfectly dried; tube dry; potential difference, 5000 volts.

Frequency.	Double deflection.	Frequency.	Double deflection
4.0	14.66	22.4	10.04
11.7	12.18	16.0	9.46
18.0	10.72	27.8	9.10

RELATION BETWEEN SPEED AND DEFLECTION.

Varnished glass, perfectly dry.

Frequency.	Double deflection.	Frequency.	Double deflection.
18.8	31.33	8.0	31.55
26.5	31.54	18.8	31.65
26.5 (another set)	31.58		

It is thus clear that the effect of varying the frequency of cell rotation is almost without influence on very dry glass, but that the effect on imperfectly dried glass is very marked. The variation in the latter case has all the regularity of a definite physical phenomenon, but so far as the properties of glass are concerned, it is wholly without meaning.

SULPHUR.

Four samples of sulphur of a high degree of purity were prepared by the method described in a paper on the "Electric Properties of Pure Sulphur" (*Phil. Trans.*, as yet unpublished). Briefly, the method consists in filtering the very pure sulphur obtained by the Chance-Claus process; distilling and finally exhausting *in vacuo*. (In the present case the exhaustion *in vacuo* was omitted.)

Spheroid A. — Perfectly annealed sulphur, of the variety which I have ventured to describe (*loc. cit.*) as "aged monoclinic sulphur," *i.e.* sulphur which preserves the melting point and physical properties of monoclinic sulphur, but which has lost the crystallographic properties of that variety.

Longer diameter	2.664 cm.
Shorter diameters	0.976 cm.
Weight	2.5625 g.
Volume	1.2945 c.c.
Density	1.9.

The proper density is 1.98, so that there was probably a small cavity in the spheroid.

Specific inductive capacity . . . . 3.162 (*loc. cit.*).

These data give a screening factor of about 0.44. This spheroid was tested by complete sets of observations in glass, mica, and ebonite guard tubes, at high and low frequencies of rotation. The weather was dry, and everything insulating well, including the spheroid, which was specially tested, but no drying material was employed.

It was found that the apparent loss, under these circumstances, depended very much on the speed of cell rotation. Thus, at a frequency of 21.9, the deflection with a potential difference of 5000 volts was 151′, and with a frequency of 4, under similar conditions, it rose to 270′. Complete sets of observations of variation of deflection with speed were made both in glass and in ebonite, but in view of what has been already said on this head, there is no point in reproducing them.

The final result, at a frequency of 22 and a temperature of 25.2° C., was

$$W = 0.0139\, F^{1.91},$$

$$\text{Percentage loss} = 11.05\, F^{-0.09}.$$

An index of 2 would not satisfy the observations, which were very regular. The sulphur was lightly paraffined during the tests from which the above result is deduced; but it was found, by comparing these with other tests of observations made before the paraffining was done, that the paraffin had no effect.

Spheroid C. — Similar in all respects to spheroid A, except that it was perfectly dried. This had the effect of reducing the deflection so much that it could not be measured with sufficient exactness to construct a curve of any pretensions to accuracy. For instance, with a potential difference of 4000 volts, the double deflections in centimeters were spheroid A, 11, and spheroid C, 2.2. The results, such as they were, pointed to an index of F of about 1.5.

After some ineffectual attempts to measure the loss with other ellipsoids, a complete series of experiments was made with —

Spheroid K. — Although it had been annealed, an analysis of a test piece showed that it contained about 7 per cent of insoluble sulphur. As this spheroid has been mentioned in the earlier part of the paper, I will merely repeat that the result at about 18 rotations per second was

$$W = 0.00048\, F^{1.857} \text{ at } 17.8° \text{ C.}$$

The specific inductive capacity being somewhat low (viz., say 3.2), this leads to a very small percentage loss, viz. 0.0038 per cent at $F = 1$; and the law of percentage loss is

$$\text{Percentage loss} = 0.00377\, F^{-0.143}.$$

As the other samples of sulphur appeared to show a very considerable variation of loss as the frequency varied, this was now made the subject of an elaborate inquiry. It was found, after a good many tests made in air of varied humidity, that the phenomenon of variation of loss with variation of frequency was a property of the sulphur and not of the apparatus; but it was much smaller than when the sulphur had not been specially dried.

VARIATION OF DEFLECTION WITH FREQUENCY OF CELL ROTATION.

Potential difference, 3500.

Frequency of cell rotation.	Double deflection in centimeters at 96 cm. scale distance.
30	16.47
18	14.00

This result differs from that obtained for spheroid A, in that the loss appears to increase as the speed increases, but spheroid A might have been slightly affected by dampness. Space will not permit of the detailed description of the series of experiments leading to the above statement for spheroid K, but it is necessary to say that the matter was thoroughly tested on many occasions, and always with the above result. One very good way of making a test is to alter the speed of rotation while the voltage is maintained constant. As this could be done by myself from my seat at the dynamometer scale, I was able to increase or decrease the deflection at will, and thus assure myself of the fact. The above numerical estimate is, of course, based on sets of observations made with the cell rotating in both directions.

PARAFFIN.

Spheroid A. — Made out of paraffin melting at about 50° C. This spheroid was tested by a thread which had been used satisfactorily for an ebonite spheroid of the same size. The resulting deflections were so small that no certain conclusion could be drawn from them.

Spheroid B. — Made out of paraffin of melting point 52° (Field's hardest) by turning.

Longer diameter	2.64 cm.
Shorter diameters	0.914 cm.
Volume calculated	1.170 c.c.
Specific inductive capacity assumed	2.3
Screening factor	0.3556

On testing this without phosphorus pentoxide a tolerable curve was obtained with an index of F of 1.38. The effect did not depend on the speed of rotation of the cell. After prolonged drying the spheroid was examined by means of a very fine fiber having a coefficient of torsion only one sixteenth of that of the fiber used for ebonite, glass, etc. The observations were not very satisfactory owing to unsteadiness of zero; however, they plotted out to a very straight line, making the index of $F = 1.95$, at a temperature of 13°.3 C., instead of 1.38 which was got when the spheroid was not artificially dried. The loss of energy was very small indeed.

It was obviously necessary to make fresh observations on a larger spheroid with smaller internal screening; consequently —

Spheroid C was tested. This was cast in a zinc mold from the same paraffin as spheroid B. The paraffin had been kept melted for some time in great excess, and the spheroid was cast by decanting off the upper layer of the melted paraffin.

Dimensions:

Longer diameter (and axis of suspension) . . .	1.80 cm.
Shorter diameters	1.38 cm.
Volume	1.802 c.c.
Internal screening factor	0.658

Observations were made at a temperature of 12°.5 C. — two complete sets at frequencies of 12.5 and 27.3, besides two subsidiary sets on days when the air was in a different hygrometric state, for purposes of confirmation. The observations were satisfactory in spite of great zero changes at both speeds; and the effect was reduced by raising the frequency. Thus, with a frequency of 12.5 and a potential difference of 6000, the double deflection was 3.28; and with a frequency of 27.3 and a potential difference of 6000, the double deflection was 2.92. This was confirmed by direct observation, viz. by changing the speed of cell rotation and noting the effect on the light spot. The logarithmic curves at the two speeds were exactly parallel, and gave 1.56 as the index of F.

The final result was:

At a frequency of 12.5 and temperature 12°.5 C.,

$$W = 0.00138\ F^{1.56}.$$

At a frequency of 27.3 and temperature 12°.5 C.,

$$W = 0.0008\ F^{1.56}.$$

The percentage losses at $F = 1$ are:

At a frequency of 12.5, percentage loss per cycle $= 1.51\ F^{-0.44}$.
At a frequency of 27.3, percentage loss per cycle $= 0.88\ F^{-0.44}$.

Various spheres of paraffin were also tested in making a comparison between the effect with pure paraffin and the effect with paraffin made exceedingly heterogeneous, but still insulating as a whole, by means of powdered Ceylon graphite. This will be dealt with later on; it is only necessary to mention here that the index of F obtained in these experiments was from about 1.5 to 1.6. The spheres being hung in the deflection tube while still warm, and the tube itself having been dried for a long period by plenty of phosphorus pentoxide, it was again noted that shifting of the zero was exceedingly marked with paraffin.

SELENIUM.

I have in my possession a large quantity of selenium, which I owe to the liberality of the Royal Society of London. During the last two years I have devoted a considerable time to the purification of this substance; the selenium as purchased containing about one half a per cent of impurity, chiefly glass, iron, sand, and copper. I have succeeded in effecting the purification of about one kilogram of this material, but am not quite satisfied as to the absence of oxides of selenium and of very minute traces of arsenic. I was enabled, therefore (within these limits), to examine the properties of selenium fairly closely. I have not, however, as yet been able to make a satisfactory measurement of the specific inductive capacity of the substance in its various modifications, for the conduction data are yet to be obtained. A determination of the specific inductive capacity of selenium in the vitreous — or fairly non-conducting state — by Kelvin's method (comparison of the condenser capacities by the galvanometer) gave a value of about 30 for the specific inductive capacity. The conductivity, however, is too great for this method to be really satisfactory, even when all precautions are taken. I therefore state the result with every possible reservation, and without laying any stress upon it. Some value or other is requisite to enable the energy loss to be calculated, and the experiments on this head were highly satisfactory throughout. When a final value for K is obtained, the energy loss formula can be recalculated without this part of the work requiring revision.

Selenium Ellipsoid A. — Cast from the selenium as purchased. The deflection in the rotating field was found to be of a quite new order of magnitude, and was, in fact, so large that experiments with this ellipsoid were abandoned on account of its size — it was evidently necessary to be sure the field was uniform.

Sphere B. — Diameter 0.7 cm. Weight 0.701 g. (which shows that it was probably not solid). The density of selenium in the vitreous state is said to be 4.25, while the calculated density of this sphere was only 3.9.

This sphere was made by heating the selenium (commercial) to the boiling point and then pouring it into a cold zinc mold, which was then thrown into water. The sphere was not quite non-conducting, *i.e.* it discharged a gold-leaf electroscope perceptibly in 30 seconds when held against the plate. The sphere was tested after diselectrifying and placing, while warm, in the dry tube, and also after two days' drying, when the results were practically the same. The index of the potential difference was 1.96, and the observations were so good that an index of 2 clearly would not satisfy the numbers. This was at a temperature of 14° C. The results are not reduced in the usual manner because the selenium was not pure, and because they are very similar to the next series, except that I could not detect with sphere B any change in the deflection when the frequency varied from 6 to 25.

SELENIUM SPHERE B.

Frequency of rotation	Single deflection in minutes.	Potential difference in volts.
10	693	5000
18	709	5000
25	692	5000

Selenium Sphere C. — Commercial selenium. Diameter, 0.7 cm.; weight, 0.8855 g.; apparently perfectly insulating, as tested on the electroscope. This sphere was evidently a solid sphere without bubbles, the calculated density being 4.93, while the density of vitreous selenium is given in the books at 4.28. The sphere was cooled by the mold without immersing in water, and hung by the same thread, under the same conditions as sphere B. It was tested both at once after mounting and after sixteen hours' drying by P_2O_5, which had the effect of *increasing* the deflection, showing that the tube was not quite dry initially. Both curves were practically parallel (index of first set 1.92, and of second set 1.90).

The effect on C was markedly less than on B, which conducted slightly. Thus, at 5000 volts potential difference, the deflection of

B was 709′ of arc,
and of C was 262′ of arc.

Correcting these numbers on account of the evident unsoundness of B, we get

Corrected deflection of B . . . . . 894′
Corrected deflection of C . . . . . 262′

or the loss with the uninsulating sphere B is 3.4 times as great as with the insulating sphere C.

The curves were perfectly regular in both cases, and there was no trouble from shifting of the zero. The general formula is not worth obtaining for a sphere of such impure material, but there was no doubt in this case as to the variability of the loss with varying frequency of cell rotation.

SELENIUM SPHERE C.

Frequency.	Deflection in Minutes.	Potential difference in volts.
10	361	5000
18	374	5000
25	377	5000

Selenium Sphere D. — Purified selenium. Diameter, 0.7 cm.; weight, 0.901 g.; cooled by mold only; cast from selenium at boiling point; density calculated from mass and volume, 5.02. This must mean either that the density of pure selenium is incorrectly given, or that the sphere was out of shape owing to the two halves of the mold not being sufficiently closely approximated. I did not care to risk the possibility of inducing a surface change by weighing the selenium in water.

The selenium was tested on one occasion when it was compared with sphere C; both were dried and put into a dry tube, — the air was not very dry (humidity 64 per cent), — but no effect was produced by blowing in hot air. The resulting curve was very good with an index 1.85; the insulation to electroscope test appeared practically perfect. Temperature of test, 15°.3 C. The amplitude was much the same as for sphere C. Thus the observations for both C and D at 5000 volts lie exactly on their respective curves; and both spheres were hung from same thread. The single deflections at this point are

$$\text{Sphere D} = 427, \text{ and sphere C} = 461,$$

showing that the pure selenium suffers a slightly smaller loss.

Taking $\mu = 28$ (for convenience of calculation), the result for sphere D is

$$W = 4.6\, F^{1.85};$$

energy of electrification in one direction is 1.075 ergs ($F = 1$).

Hence the loss appears to be over 400 per cent. I am not prepared to say at this stage whether such an extraordinary result can possibly be due to any error in estimating K, but I think not. The dynamometric results are above suspicion. There is, so far as I can see, no impossibility in the

loss being greater than the energy of electrification in one direction; for suppose that polarization is set up and vanishes after a time, short, but not very short compared with one revolution of the field, then the source is drawn upon for a fresh supply of energy and the process is repeated as often as we like. While the charges are distributing themselves by conduction, the angle between the line passing through the center of the sphere and the center of gravity of the charge and the direction of the field, may increase and give rise to the observed increased torque.

On the other hand, if the polarization vanishes in a time indefinitely short compared with the period of field rotation, the angle is practically always zero, so that in perfectly or even in well conducting bodies no dynamometric effect is to be expected. This, of course, is not really the physical case, for if we have a real conductor, *i.e.* one in which conduction ocours in times very short compared with the period of field rotation, then we may get perfect internal screening, and no work of polarization is done, because there is no force except within an infinitesimal distance of the boundary.

Comparing spheres C and D at 5000 volts potential difference, and correcting for the difference in weight (on hypothesis that volumes are proportional to weights), we find —

Loss in sphere D proportional to 427,
Loss in sphere C proportional to 468.

No observations on the variation of loss with varying frequency were made with this sphere.

Resin.

The resin employed consisted of picked pieces of the resin sold as a flux for soldering lead. Experiments were made on spheres of 0.7 and of 1 cm. in diameter, which were cast in zinc molds. A large number of spheres were examined under very diverse circumstances, because there was an utter want of regularity in the phenomena observed. The following comparison will give an idea of the magnitude of the effects.

Small ebonite sphere, diameter say 0.7 cm. nearly:

Deflection in minutes at 5000 volts . . . 400

Resin sphere θ, diameter say 0.7 cm.:

Deflection in minutes at 5000 volts . . . 123

Resin sphere γ, diameter say 0.7 cm.:

Deflection in minutes at 5000 volts . . . 484

The coefficient of torsion of the threads being sufficiently nearly the same.

As the resin has about the same dielectric constant as ebonite, the percentage loss in the case of sphere γ is about 18 per cent of the initial energy of electrification, when F is about 13, or the potential difference of the plates is 5000 volts.

In order to decide whether any explanation of the extraordinary irregularity of the results with resin could be accounted for by any change in the apparatus, the smaller ebonite sphere was remounted and rapidly examined. It was found to give exactly the same results as before, and the zero was perfectly steady. With the resin, on the other hand, the zero was always very unsteady, and great care was requisite to get any reliable results at all. All the spheres were cast from the same sample of resin, and on one occasion two were cast at the same time from the same lot of resin melted in a dish. These two were widely different in their properties. It may be added that owing to the practice of casting the resin in a nearly cold mold so as to avoid the sticking of the resin to the sides of the mold, the resin spheres were always in a state of strain, and generally cracked considerably within twenty-four hours after casting.

The following table will show the uselessness of going into the question of dielectric losses in resin in any detail; it represents but one set of experiments out of many made on this subject. All the experiments here included were made on one day, and under exactly similar conditions; this, of course, means that in this case there is no prolonged drying. The resulting curves (as might be expected from the difficulty of observing) were very irregular as compared with those obtained from other substances, and, therefore, too much stress must not be laid on the value assigned to the index. A typical set of numbers will be given for one sphere, from which the degree of certitude of the result may be, to some extent, inferred. It must not be forgotten, however, that when the zero is subject to a considerable drift, the method of observing by reversing the direction of rotation will not eliminate the drifting, unless the observations are made under comparable conditions as to the time elapsing between the raising of the potential difference to the required point and the taking of the reading. This is not quite easy to do, because the observations of any potential difference consist of the reading of ten elongations (five on each side of the resting point), and difficulties in the way of keeping the potential difference steady often cause a small difference in the time required for the observations.

The effect of increasing the speed of rotation with resin was to increase the deflection: at the same time, the effect was not large. Thus, in the case of a sphere freshly cast for the purpose of a test, the deflections increased from 20.3 to 21.8 scale divisions as the frequency increased from 10 to 25. The creep of the zero was, however, so marked that it is quite

possible to account for the variation observed by supposing that the error introduced into the result and remaining uneliminated on reversal of the cell, was the quantity affected by the change of frequency.

RESIN SPHERES.

Deflections and potential difference.

Sphere.	Diameter of sphere.	Rotations in minutes of arc with a potential difference of 5000 volts.	Index of F.	Remarks.
β	1.0 cm.	236	2.4	Dried two days.
γ	0.70	489	2.8	Dry in dry tube; taken at once.
δ	1.0	172	2.2	Resin re-heated; observed at once.
ζ	1.0	440	1.86	Same melt as last; observed at once.
θ	0.70	123	2.23	Same melt as last; observed at once.
χ (Resin and graphite)	0.70	442	2.02	Same melt as last; 10 per cent by volume of fine graphite added.

TABLE SHOWING RELATION OF POTENTIAL DIFFERENCE AND DEFLECTION.

Resin sphere γ, 0.7 cm. diameter.

Potential difference volts.	Single rotations of the sphere in minutes of arc.
2000	39
3000	122
4000	248
5000	484

In drawing the logarithmic "plot," weight is given to these observations above that assigned to the other two.

A good many other observations which were made on resin spheres resembled those quoted in giving an index for F from about 2.2 to 2.3, but as threads of varying stiffness were used, and as the general results exactly resembled those quoted, there is no reason for reproducing them. I am unable to see any theoretical objection to the index being higher than 2, for if the effect depends in some measure on internal conduction, we know that this factor will itself depend on the potential difference, since substances

like resin are far from obeying Ohm's law. As in the case of selenium, which also gave very anomalous results, the resin appeared to insulate perfectly when tested by the gold-leaf electroscope.

DIELECTRIC LOSS IN HETEROGENEOUS SUBSTANCES.

These experiments will be described in the order in which they were made. The intention was to examine the properties of some homogeneous substances, and to compare these properties with those exhibited by the same material when it was purposely rendered heterogeneous by mixing it with some inert but conducting substance. Resin spheres could be cast so as to be perfectly transparent, and it was considered that this transparency would be a guarantee of, at all events, the first order of homogeneity. No doubt the strains set up on cooling introduce a heterogeneity of a certain kind, but it must, from the electrical point of view, be of a single kind only.

The complete and uniform transparency of the substance negatives the possibility of any sudden or appreciable change of the electric or magnetic constants, unless it be assumed that these quantities vary inversely — a theory very far removed from any known facts. The chemical composition must be uniform, because the melted resin is well stirred and cools as soon as it is poured into the mold; the viscosity also is very great. These facts negative any hypothesis of gravimetric separation. The resin does not crystallize under any circumstances, so far as I am aware, and consequently the only form of heterogeneity that can be invoked is such as results from the unequal distribution of strain. It is just possible that the specific resistance may be a function of the strains, and if so, the necessary heterogeneity would be introduced. I know of no facts in support of this theory, except, perhaps, my own observations on sulphur, which showed that the specific resistance is an exceedingly unstable property, and one which is specially sensitive to minute changes of chemical composition. On the other hand, I found that crystalline sulphur may pass from the monoclinic state into a second state, in which the crystallographic properties are entirely lost, without the resistance being in any way affected, *i.e.* the specific resistance under specified conditions as to voltage, etc., may remain above 10^{28} — my limit of discrimination.

On the whole, therefore, it appears —

That the heterogeneity which is required by Maxwell's theory to account for the observed dissipation of energy in clear resin spheres is, *so far as is known*, entirely absent.

In order to test the Maxwell theory, therefore, it appeared to be necessary to introduce, purposely, a degree of heterogeneity indefinitely great as

compared with anything naturally existing in clear resin. The kind of heterogeneity required is such as to make the product specific resistance × specific inductive capacity vary from point to point. Now, as I have said, the "specific resistance" of such a substance as resin does not exist for the substance in bulk, but it may be supposed that it exists for exceedingly small portions of the substance, *i.e.* though a plate of resin does not obey Ohm's law, a particle may do so. This is the assumption made above. It is by no means easy to arrange an experiment in which we can be sure that the heterogeneity is greater than that due to the pure substance, say, resin, for we have practically only two courses open to us: we can introduce a conductor and distribute it through the mass, or we can introduce an insulator with a value of K differing from that of the substance. In the former case, we are in the dark as to the specific inductive capacity of the substance added. For instance, I added graphite, which conducts, say, 10^{14} times better than resin for electric intensities such as were employed. This estimate, of course, is very rough. Is it possible that the specific inductive capacity of graphite can be greater than that of resin in this ratio? On the other hand, if an insulator, such as powdered flint glass, be added, we have, say, a threefold change of specific inductive capacity, and, perhaps, a corresponding change in resistance; for it may be urged that we cannot argue as to the resistance of small particles from the resistance observed in plates; especially when, as in the present case, the actual resistances are practically beyond our powers of discrimination. Unless there is some physical law connecting the relation of specific inductive capacity with resistance, the probabilities are all in favor of a heterogeneity of the kind aimed at being introduced by the addition of graphite to resin.

As was stated at the commencement of the paper, therefore we cannot hope to settle the question of the theory of heterogeneity however we try to vary the constitution of the substance examined; we can only hope to raise a probability one way or the other.

My first experiments were made on mixtures of resin and graphite, the latter obtained by grinding Ceylon graphite, of a kind that conducted very well, and passing it through a sieve having 100 threads to the inch. My first experiment was made with a sphere which contained so much graphite that it conducted as a whole, and with it no deflection was produced by rotating the field. The same result was obtained with aluminium, carbon, and tinfoil-coated paraffin ellipsoids. On reducing the quantity of resin to say 20 per cent by volume (it was not measured), I obtained a perfectly black sphere, which when broken had a dull fracture, but which did not discharge an electroscope much faster than a pure resin sphere.

I compared the graphite-resin and the resin sphere, which were made

from the same melt of resin, were of the same size, and hung by the same thread. In this comparison the indexes appeared about the same, viz. 1.9, though the results with the resin were very irregular, but the effect was greater in the graphite sphere than in the sphere made of resin only. At a potential difference of 6000 volts, the deflection of the graphite sphere was 128′ and of the resin sphere 25′—an increase in the ratio of 5 to 1.

It was then found that resin is itself exceedingly variable, as shown under that head.

The variability of resin having been ascertained, a fresh trial was made by adding 10 per cent by volume of fine graphite to the resin. The result is indicated in the table of results with resin spheres. The effect of the addition of graphite here appears as practically nil. The curve of potential difference and deflection was quite regular, and the observations at least as satisfactory as with the pure resin.

As M. Hess has given reasons (*loc. cit.*) for thinking that the dielectric loss should be a function of the speed, the variation of deflection with varying rapidity of cell rotation was very carefully studied, with the following result:

GRAPHITE-RESIN χ.

Table showing variation of deflection with varying frequency.

Frequency.	Double deflection in scale centimeters.
10	40.0
18	47.5
28	48.5

This result was obtained in two repetitions of the experiment, and is not to be explained by any assumed moisture effect.

As an additional precaution, the ebonite sphere, formerly studied under very various circumstances, was remounted in place of the resin-graphite, and gave identically the same results as before.

In order to observe whether the deflection speed relation depends on the resin or on the graphite, a new resin sphere was mounted and examined. This sphere was cast from the same sample of resin as the resin-graphite χ. The results were as follows:

Frequency.	Double deflection.
10	40.6
25	43.6

It was clearly desirable to make use of some material giving more uniform results than resin, if such could be found, and, more important still, less affected by the tendency of the zero to shift during observation. To these must be added the qualifications already discussed, as to homogeneity,

etc. Practically the only available substance appeared to be paraffin; and even this cannot be prevented from crystallizing to some extent. However, by pouring the paraffin, heated only just above the melting point, into a very cold mold, it was found possible to obtain fairly transparent spheres. On the other hand, paraffin was the substance which, next to resin, had given the greatest amount of trouble by shifting of zero. This peculiarity was found to be strongly developed in the spheres now examined; consequently, there is little more to be learned by the use of paraffin than by the use of resin, and a summary of observations on two spheres will sufficiently meet the case.

Comparison of a sphere of pure paraffin and a sphere of paraffin containing so much graphite that it was just not sensibly conducting:

Temperature, 18°.4 C.; speed, 16 revolutions per second; humidity, 52 per cent; tube, etc., perfectly dry.

Index of F, graphite sphere = 2.

Index of F, pure paraffin sphere, 1.6 to 1.9. Observations unsatisfactory, owing to zero changes.

Deflections, in minutes, of the spheres at 3000 volts potential difference:

Paraffin sphere, 162
Paraffin and graphite sphere, 290
} same thread used in both cases.

The paraffin-graphite sphere, therefore, experiences an energy loss 1.8 times as great as the loss in the paraffin sphere.

On varying the speed of rotation, the loss with graphite-paraffin was found to increase considerably, as follows:

Speed of rotation.	Double deflection in scale divisions.
10	29.2
16	30.0
29	34.8

The pure paraffin sphere gave a similar but rather smaller effect.

Effect on the index of F of the addition of graphite to resin and paraffin:

Resin-graphite α	Index = 2.30	Perhaps affected by a damp tube; not much weight.
Resin graphite β	Index = 1.89	Quite dry; nothing known against it.
Resin-graphite χ	Index = 2.02	Quite dry; nothing known against it.
Paraffin-graphite A	Index = 2.00	Quite dry; nothing known against it.

The index thus shows a tendency towards the value 2.

In all cases the effect of a change of speed on heterogeneous material was to give higher values of the deflection with increasing speeds.

The general result may be stated thus: The addition of graphite, either to resin or paraffin, tends to increase the loss and to bring the index towards 2. In the case of resin the change of index is not well marked because the index for resin alone is quite erratic and is already in the mean not far from 2. It is impossible to attempt to calculate the law of loss satisfactorily on the Maxwell theory, as has been done by Hess (*L'Éclairage Électrique*, Vol. 7, 450), because Ohm's law, or some analogous assumption, has to be made and applied to dielectrics. It is a mere case of arguing in a circle to say that "were it not for electric absorption these substances would appear to obey Ohm's law — as the particles in reality do." On the other hand, non-conducting liquids, as examined by Thomson and Newall (*Phil. Proc.*, 1886), appeared to obey Ohm's law, but the elaborate investigations of Köller lead to a totally opposite result. Samples of sulphur containing both soluble and insoluble sulphur examined by me gave exceedingly small residual charge effects, and yet conducted appreciably. They were, however, very far indeed from obeying Ohm's law; in fact, the conduction was not even continuous. A great deal is known about the conduction of very impure selenium, and this substance is not even on speaking terms with Ohm's law, as the researches of Werner Siemens and others have shown. The evidence, therefore, so far as is known, is that substances such as I have examined do not obey Ohm's law in bulk, and till further information is to hand it must be assumed that they do not do so either when reduced to elements of volume. Consequently we cannot hope to test the Maxwell theory by a study of the index of F in the energy-loss equation.

Now as to the magnitude of the increase of loss produced by adding graphite. This is quite inconsiderable in comparison with the enormous increase of heterogeneity which must in all probability (neglecting wild hypotheses) have been introduced. If the original loss, say in paraffin, were produced by heterogeneity, one would certainly have expected that the loss would have been much more than doubled when graphite was added to the verge of conductivity, but, as a matter of fact, the loss was not doubled.

No one doubts, however, that heterogeneity can give rise to a loss of energy, and the question may be put whether the observations recorded above are not such as may be most naturally explained by admitting the possibility of dielectric loss as due to some kind of hysteresis, and supposing that the additional loss introduced when the material is made heterogeneous is a loss to be accounted for on Maxwell's theory, which simply adds on to the normal hysteresis loss. But electric hysteresis, in the sense in which I use the word, may be either static or viscous. The experiments described, however, show that the viscosity, if it exists, is of such a nature

as to produce a hardly appreciable effect when the frequency varies from 5 to 30 per second, but, as will be shown presently, is sufficient to obliterate the phenomenon entirely at a frequency of several millions per second.

Experiments on Liquids.

Thomson and Newall (*Phil. Proc.*, 1886) thought they detected a trace of residual charge effect in bisulphide of carbon (*loc. cit.*), and I therefore considered it worth while to see if I could set up a rotation in this liquid. For this purpose a thin flint glass tube was mounted on the stand and dipped into the rotating field in exactly the same position as was habitually occupied by the guard tube. I tried a good many ways of indicating rotation: the most satisfactory of these was to rotate the field, then draw up the tube, and throw some very fine alumina into the liquid. Owing to the low rate at which this substance subsided, its motion could be accurately observed. The liquids examined were carbon bisulphide, benzene (C_6H_6), kerosene (from a fresh tin opened for this purpose), and distilled water. The carbon bisulphide was tested both highly purified and with sulphur in solution. The benzene had been crystallized. In no case could I find the slightest trace of rotation, though the effect of slightly turning the tube was instantly observed. The individual particles of alumina were also caused to spin violently about their own axes. The test was, in fact, a very delicate one, and the result is quite conclusive. It is to be noted that the liquids employed have by no means the same specific inductive capacity as flint glass, so that the result is not to be explained by any assumed want of discontinuity at the boundary.

Dielectric Losses in very rapidly Rotating Fields.

The ordinary electromagnetic explanation of the opacity in such substances as black pitch, bitumen, or ebonite, which, nevertheless, insulate well in bulk, is that the conductivity for small electric intensities, and over small elements of volume, may be considerable. This is, of course, not the only explanation; any cause which leads to a dissipation of electromagnetic energy might be equally cited, and amongst the most prominent of these is the possibility of electric or magnetic "hysteresis" loss. With regard to the latter, its investigation is at present beyond our powers in cases where the permeability approaches unity. I have myself, with the aid of my students, spent more than three years in trying to obtain a permeability curve for bismuth for small magnetic forces — an investigation which resulted simply in defeat. I think it quite possible, however, that experiments by a rotating magnetic field might succeed in bringing mag-

netic hysteresis losses to light in ordinary dielectrics — an investigation which would be easier in some respects than that with which we are now dealing, for we could reap the full advantage of the properties of quartz fibers without having to separate the effects observed from those due to accidental electrifications. With regard to electric hysteresis, however, we are more favorably situated, in consequence of the dielectric constants of ordinary transparent media being higher than the corresponding magnetic quantities. In view, however, of the effect which speed of field rotation has upon the dielectric loss, at low speeds, it is necessary to raise the speed of rotation to something more comparable with the frequencies met with in optical problems.

It has been already stated that none of the substances examined by me exhibited any dielectric loss at all at very high frequencies, and this negative result requires me to show that my arrangements were such that a loss of energy could not have been overlooked had it existed to any appreciable extent.

The only practicable method of obtaining a very rapidly rotating field is by the harmonic composition of oscillating electric forces of high frequency, and the most convenient mode of obtaining the latter is to employ oscillatory currents.

As a source of such currents I used the apparatus devised by Ebert (Wiedemann's *Annalen*, 53, 144, August, 1894). As this apparatus was exactly copied, — the author giving very full particulars, — I will merely state that by its aid it is possible to obtain oscillatory currents making about 7×10^6 complete cycles per second (*loc. cit.*, p. 152), having a wave length in air of 42 meters and a logarithmic decrement of about 5×10^{-1}. This leads to the completion of some 2000 oscillations before the amplitude falls to $\frac{1}{e}\left(=\frac{1}{2.7}\right)$ of the initial value. No attempt was made by me to verify Ebert's numbers, which may, I think, be safely taken, otherwise than by roughly calculating values for the capacity and self-induction as actually employed by me, and noting that they led with as much accuracy as could be expected to the same values as those just quoted. The diagram of Ebert's arrangement of Tesla's apparatus is included in the general diagram of connections, Fig. 3.

It was necessary for me to feed the oscillatory spark system much more vigorously than was done by Ebert. This led to the following modification: Instead of using a large Wimshurst machine, I used a very large induction coil which was formerly in the possession of the late J. E. H. Gordon, and which is profusely illustrated in his well-known work on electricity. The hammer break of this coil was replaced by a "rapid break," consisting of a disk of slate some six inches in diameter, provided

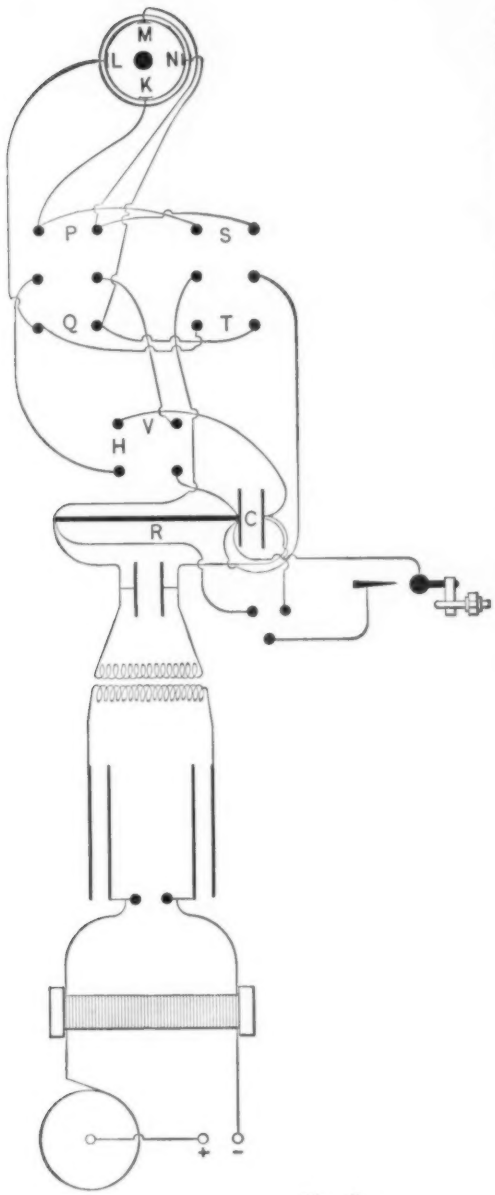


Fig. 3.

Diagram showing arrangement of Ebert apparatus, and connections between the Ebert apparatus, the phasing resistance and capacity, and the rotating field coil.

R. Phasing resistance.
C. Phasing capacity.
H, *V*. Key for reversing the direction of rotation.
P, *Q* and *S*, *T*. Two six-point keys for interchanging the connections between the armatures and phasing terminals.

with thirty copper segments against which a carbon brush was carefully adjusted. The rotating break was driven by a fan-governed water motor and provided with a contact pin enabling its speed of rotation to be measured by means of the electromagnetic apparatus of the myograph. A contact breaker requires to be very well made if it is to work regularly, and this was done in the present case. I generally used about 300 sparks per second between the primary discharge balls of the Ebert apparatus, which were 3 cm. in diameter and from 1 to 1.3 cm. apart by the shortest path. A torrent of sparks of this frequency will not give rise to oscillations in still air, but by blowing a strong air blast from a Root's blower across the spark gap, each spark becomes oscillatory. This was tested by examining the discharge by a rotating mirror, and also by examining the discharge at the secondary terminals of the Ebert appliance in like manner. The discharge of the Ebert apparatus was very vigorous, and it was necessary to round all corners of the conductors involved and to coat them thickly with paraffin.

The rotating field was set up inside a glass tube by means of four narrow armatures. At first the armatures were made of thin copper, but the Tesla flames and other effects at the edges led to their being replaced by carefully rounded pieces of aluminium, produced by flat-drawing a thick bit of aluminium wire. The armatures were coated by dipping in paraffin to a depth of about one millimeter.

The glass tube carrying the armatures formed the guard tube of the suspended system, and had a short wide side connection to a flask containing phosphorus pentoxide. The suspended apparatus required to be carefully screened as in the experiments formerly described.

There is considerable difficulty in obtaining a suitable resistance to combine with the condenser in the combination necessary to obtain the required phase difference between the potential differences of the armatures.

The condenser must be small compared with the condenser of the Ebert apparatus, or the theory of the experiment becomes unmanageable. The capacity of the Ebert appliance likewise cannot be large, or the damping becomes too great, and also the oscillations become slower. A small capacity in the phasing apparatus, however, implies a large resistance, if the potential difference at the resistance terminals is to be equal to that at the terminals of the condenser — a necessary condition. Strips of wood, graphite, etc., were tried without effect, the former exhibiting a variable resistance, the latter burning away when in sufficiently thin streaks, and hence being even more variable. I was finally compelled to use a pure water resistance. This consisted of a U tube with the two arms close together to diminish inductance; the tube was inverted thus, ∩, and the free ends dipped into two tubes full of mercury. By raising or lowering

the U tube, which had a small reservoir at its upper end, the resistance could be varied. In most of my experiments a tube of about 2 mm. bore and about 20 cm. long, measured from electrode to electrode, was employed. The phasing condenser plates were 15 cm. in diameter and several centimeters apart, and were adjustable by sliding.

The equality of the maximum potential differences of the phasing combination were ascertained by a spark micrometer, a point and ball being used when it was a question of adjusting to equality, and two small spheres when the potential difference was measured. This micrometer could be thrown into either position by means of a key. In addition to this key there were three others. The first, *HV*, was used to reverse the connections of the condenser to the armatures with which it happened to be associated. The two six-point keys *PQ*, *ST*, enabled the armatures to be interchanged with respect to the terminals of the phasing apparatus. This was necessary, because the dielectric could not be centered with perfect certainty and, consequently, tended to become electrified. When this happened, small deflections made their appearance, the connection of which with the phenomenon sought required to be ascertained.

Thus the armatures *M* and *K* could be connected to the condenser, and by working the key *HV* the direction of rotation could be made either clockwise or the reverse at pleasure. The armatures *K* and *L* could then be placed upon the condenser and two fresh observations made. The substance to be examined was generally in the form of a long cylinder and was suspended by as fine a fiber as possible. The deflections were read by means of a scale at a distance of 150 cm.

It was necessary for me to have a means of estimating the armature potential differences from the observations at the spark micrometer. This would have been easy in view of Baille's and other results on spark length in air, were it not for the fact observed by Jaumann (*Wien. Bericht*, 97, p. 765), that the spark length is greatly affected by small oscillations of electric force at the surface of the conductors. Jaumann, however, does not give any rule by which the magnitude of this effect may be ascertained in any particular case such as the present, and it therefore became desirable to express the armature potential differences in terms of the potential difference of the primary spark gap in the Ebert apparatus. I was so fortunate as to secure the assistance of my friend, Professor Lyle of Melbourne, in this investigation, of which, however, I only propose to give a sketch.

Let $L = M = N$, the self and mutual inductions of the Ebert coils being numerically about 6×10^4 C.G.S.

Let $R = S$ — the resistances of the coils (about 0.088 ohm).

Let two similar circuits be arranged as in Fig. 4.

When the spark bursts across the gap in the primary circuit, consider the resistance of the gap during the oscillations as zero.

Let R be the resistance, and L the self-induction of each circuit; J the capacity of each condenser; x the charge on the primary, and y that on the secondary condenser at any moment. We have then the following equations:

$$L(\ddot{x}+\ddot{y})+R\dot{x}+\frac{x}{J}=0,$$

$$L(\ddot{x}+\ddot{y})+R\dot{y}+\frac{y}{J}=0.$$

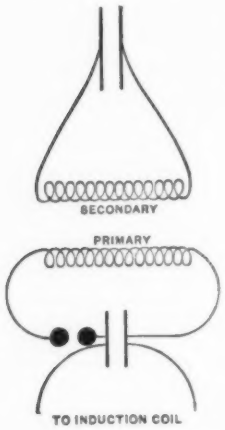


Fig. 4.

A solution of these equations is

$$y=e^{-\frac{R}{4L}t}\,(A\cos\mu t \text{ and } B\sin\mu t)-\frac{x_0}{2}e^{-\frac{t}{JR}},$$

where $$\mu^2=\frac{1}{2JL}-\frac{R^2}{16L^2},$$

and x_0 is the charge on the primary condenser when the knobs in the primary circuit are on the point of sparking.

The initial conditions

$$t=0,\ x=x_0 \text{ gives } A=\frac{x}{2},$$

$$t=0,\ y=0 \text{ gives } B=0,$$

so $$y=\frac{x_0}{2}\left(e^{-\frac{R}{4L}t}\cos\mu t-e^{-\frac{t}{JR}}\right).$$

The second term, $e^{-\frac{t}{JR}}$, may be neglected, as it very quickly vanishes. We may take then the maximum value of y as $\frac{x_0}{2}$. The maximum value of the potential difference of the plates of the secondary condenser is therefore one half the sparking potential difference of the knobs in the primary circuit. That this is the case was verified by measuring the sparking distance between knobs. Thus, on one occasion the primary potential difference was ascertained by Baille's table to be 105, and the secondary 52 electrostatic units.

Putting $\lambda=\frac{R}{4L}$, we may write $y=\frac{x_0}{2}e^{-\lambda t}\cos\mu t$.

The values of the constants are:

$$R=0.088 \text{ ohm},$$

$$L=6\times10^4 \text{ electromagnetic units},$$

$$J=1.5\times10^{-10} \text{ electromagnetic units};$$

so $\lambda=366$, $\mu=2.4\times10^7$.

In the above the capacity of the condenser used to get the phase difference is neglected, as also the capacity of the armatures.

Now consider the secondary condenser as an origin of electromotive force given by the formula,

$$E = E_0 e^{-\lambda t} \cos \mu t,$$

where $E_0 = \frac{x_0}{2f} = \frac{1}{2}$, the sparking potential difference of the knobs in the primary circuit.

Let the potentials at any instant be as marked on Fig. 5.

Assume that there is only one resistance large enough to be considered, and no inductances.

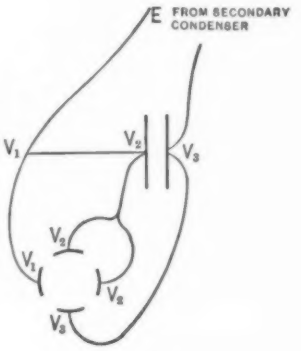


Fig. 5.

$$V_1 - V_3 = E = E_0 e^{-\lambda t} \cos \mu t.$$

$V_1 - V_2 = V\dot{q}$, $V_2 - V_3 = kq$ } where q is the charge on small condenser and $k = \frac{1}{\text{capacity}}$ of small condenser.

Hence, $$r\dot{q} + kq = E_0 e^{-\lambda t} \cos \mu t.$$

Let $$q = e^{-\lambda t}.$$

$$\dot{q} = e^{-\lambda t}(\dot{p} - \lambda p);$$

so $$r\dot{p} + (k - r\lambda)\, p = E_0 \cos \mu t,$$

the solution of which may be written:

$$p = \frac{E_0}{k - r\lambda} \cos \phi \cos (\mu t - \phi),$$

where $$\tan \phi = \frac{\mu r}{k - r\lambda}.$$

Hence, $$q = \frac{E_0}{k - r\lambda} \cos \phi e^{-\lambda t} \cos (\mu t - \phi),$$

$$\dot{q} = -\frac{E_0}{k - r\lambda} \cos \phi e^{-\lambda t} \{\lambda \cos (\mu t - \phi) + \mu \sin (\mu t - \phi)\},$$

$$\dot{q} = -\frac{E_0}{k - r\lambda} \cos \phi \sqrt{(\lambda^2 + \mu^2)} e^{-\lambda t} \cos (\mu t - \phi - \psi),$$

where $$\tan \psi = \frac{\mu}{\lambda}.$$

With the values of μ and λ given, $\frac{\mu}{\lambda} = \frac{2.4 \times 10^7}{366}$, which may be considered ∞, and so $\psi = \frac{\pi}{2}$.[1] We may therefore write

$$V_2 - V_1 = -r\dot{q} = \frac{E_0}{k - r\lambda} r\mu \cos\phi e^{-\lambda t} \sin(\mu t - \phi),$$

$$V_2 - V_3 = kq = \frac{E_0}{k - r\lambda} k \cos\phi e^{-\lambda t} \cos(\mu t - \phi).$$

The maximum values of these quantities may be considered as

$$\frac{E_0}{k - r\lambda} r\mu \cos\phi, \text{ and } \frac{E_0}{k - r\lambda} k \cos\phi.$$

Experimentally either k or r can be altered until these values are equal, which occurs when $k = \mu r$.

Now $\tan\phi = \frac{\mu r}{k - r\lambda}$, so the condition $k = \mu r$ makes $\tan\phi = 1$ and $\cos\phi = \frac{1}{\sqrt{2}}$.

We have now $$V_2 - V_1 = \frac{E_0}{\sqrt{2}} e^{-\lambda t} \sin(\mu t - \phi),$$

$$V_2 - V_3 = \frac{E_0}{\sqrt{2}} e^{-\lambda t} \cos(\mu t - \phi).$$

The maximum values of these quantities may be taken as $\frac{E_0}{\sqrt{2}}$.

So the maximum potential difference across the armatures should be 0.353 times the sparking potential difference of the knobs in the primary circuit.

[*To be concluded.*]

[1] In Recent Researches in Electricity and Magnetism, p. 295, Professor J. J. Thomson gives as the value of the impedance of a straight cable for very rapid currents

$$\left(\frac{\sigma\mu p}{2\pi a^2}\right)^{\frac{1}{2}} + \left(\frac{\sigma'\mu' p}{2\pi b^2}\right)^{\frac{1}{2}}.$$

The cable is formed of two concentric conducting cylinders separated by a dielectric. The outer radius of the core is a, and the inner radius of the inclosing cylinder, b. σ and σ' are the respective specific resistances of the conductors, μ and μ' their respective magnetic permeabilities, $\frac{p}{2\pi}$ is the frequency.

Suppose we consider that R, the resistance of either coil for steady currents in the above calculation, is replaced by $\left(\frac{\sigma\mu p}{2\pi a^2}\right)^{\frac{1}{2}} \times l$, where l is the length of the wire, in this case 1000 cm. Taking the frequency as 1×10^6 per second, the value of λ would be about 900 instead of 366; the value of μ would be practically unaltered, so $\frac{\mu}{\lambda}$ could still be considered ∞ and $\psi = \frac{\pi}{2}$.

MINOR CONTRIBUTION.

The Distribution of Alternating Currents in Cylindrical Wires.

By Ernest Merritt.

WHEN an alternating current flows along a wire the distribution of current is not uniform throughout the cross-section, as it would be in the case of a steady current. Experiment and theory both show that with alternating currents the current density is always greatest at the surface. As the frequency of alternation is increased, the difference between the current density at the surface of the wire and that at points in the interior becomes more marked, and finally, when frequencies are reached of the order of Hertz' vibrations, the current is confined to an extremely thin layer on the surface. In such cases the conductivity depends rather upon the surface of the conductor than upon its cross-section.

This so-called throttling effect has been known for many years. The fact that such phenomena might occur was indeed suggested by Maxwell. Attention was attracted to the subject by the experiments of Hughes on self-induction, and its theory has been very completely worked out by Heaviside, Rayleigh, Kelvin, and J. J. Thomson. The conclusions have also been verified, at least from a qualitative standpoint, by the experiments of Hertz, Lodge, and others.

The general interest which this phenomenon of throttling has aroused needs no better evidence than the mention of the names of the eminent men who have devoted their time to its investigation. It is, in fact, evident that the phenomenon is from several points of view an important one. In the first place, it affords an excellent illustration of the manner in which electrical disturbances are propagated, and accentuates the fact that such disturbances proceed from the ether into the wire, instead of being transmitted by the conductor itself. On the other hand, as soon as frequencies in excess of a few hundred periods per second are reached, the fact that the whole cross-section of the conductor is not utilized in the conduction of alternating currents has important practical bearings. In the case of iron wires the throttling effect, and the resulting increase in the effective

resistance offered by such wires to alternating currents, become noticeable at very low frequencies.

Although, as above stated, the theory of the subject has received complete and varied treatment, numerical computations seem to have been made in only a few cases. It is generally known that this concentration of the current at the surface of a wire will occur, but it is a matter of some difficulty to find data which will enable one to tell just how great the throttling effect will be in any specified case. I have encountered this difficulty several times during the past few years, and at length decided to make the computations necessary for plotting a few curves, so as to be able to represent the results in such a form as will give a concrete idea of the nature and magnitude of the effect. The belief that the need of such numerical results has also been felt by others must serve as the excuse for this article.

In general, the computation of the distribution of current across the cross-section of a wire is so complicated as to be almost beyond the scope of analytical treatment. In a few special cases, however, the formulas involved may be reduced to a comparatively simple form. One of these cases is that of two flat conductors, so close together that they may be treated as plane conducting sheets. This case has been discussed by Lord Kelvin. Another case, and one which approaches more nearly to practical conditions, is that of a cylindrical wire at such a distance from surrounding bodies that the magnetic force in its immediate neighborhood may be looked upon as approximately the same at all points equally distant from the wire. In this case, from the symmetry of the conditions, we are authorized to assume a symmetrical distribution of current throughout the conductor, so that if r represents the distance from the center, x the position in the wire measured in the direction of the axis from some arbitrary zero point, and t the time, the current density will be a function of these three quantities only. The condition of symmetry about the axis of the wire can be accurately reached only by making the return circuit in the form of a hollow cylinder completely surrounding the wire; but in the case of a circuit formed of two wires whose distance apart is fifteen or twenty times the radius, the condition of symmetry is approached with sufficient closeness for ordinary purposes.

In discussing the theory of the subject most writers have started with Maxwell's equations for the electro-magnetic field, have modified these to suit the conditions of symmetry about an axis, and have then written down and discussed the solution of the resulting differential equation. In presenting the subject to beginners it seems to me better not to assume Maxwell's equations as already derived, but to start from the fundamental physical laws upon which these equations themselves rest, and then to

apply these laws to the case in question. To obtain the differential equation by this method is perhaps in appearance not so direct as to make use of Maxwell's equations; but by referring back to the fundamental principles involved the physical side of the problem is likely to be better understood, and the process of solving the problem is less apt to be looked upon as a mere manipulation of symbols. The discussion which follows is intended as a brief outline of a method for solving the problem, which I think may be used to advantage with a class whose members are more familiar with physical principles than with the methods of physical mathematics.

In addition to the condition of symmetry about the axis of the wire, we have at our disposal for solving the problem of distribution of current the following three laws:

1. The line integral of the magnetic force about any conductor which carries a current is equal to 4π times the current. Or, as it is often stated, "the work done in carrying unit pole once around the current i is equal to $4\pi i$." This law, proven experimentally for the case of currents flowing in wires, may be extended, at least tentatively, to the case of currents in three dimensions. The fact that conclusions based upon this extension have been experimentally verified, justifies the assumption that the law applies. A mathematical expression of the law, with especial reference to the problem in hand, may be derived as follows:

Consider a small portion of the cross-section of the wire, bounded by arcs of circles whose radii are r and $r+dr$, and by the radial lines OA', OB'. If the current density is u, the total current flowing through this small area $AA'B'B$ is $ur\theta dr$.

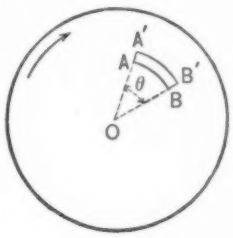


Fig. 1.

The magnetic force at A is at right angles to OA (since the lines of force are circles about O) and has the same value at all points along AB. Let the magnetic force at A be denoted by T. T is then a function of r, and its value at points along $A'B'$ will be $T+\frac{\partial T}{\partial r}dr$. The work done by the magnetic forces when unit pole is carried around the path $AA'B'B$ is therefore

$$\left(T+\frac{\partial T}{\partial r}dr\right)(r+dr)\theta - Tr\theta,$$

and this, in accordance with the law stated, is equal to 4π times the current through $AA'B'B$. We thus have

$$\left(T+\frac{\partial T}{\partial r}dr\right)(r+dr)\theta - Tr\theta = 4\pi ur\theta dr.$$

$$\therefore\ 4\pi u = \frac{\partial T}{\partial r}+\frac{1}{r}T. \qquad (1)$$

2. The line integral of the electric intensity around any circuit is equal to the rate of decrease of the magnetic induction through the circuit. Or in other words, the electromotive force induced in any circuit is equal to the rate of decrease of the flux of induction through that circuit. This law also has received direct experimental verification only in the case of circuits of finite magnitude, but its application has been extended, tentatively as before, to the case of infinitely small circuits.

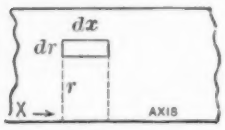


Fig. 2.

To apply it to the present case, consider a section of the wire formed by a plane passing through the axis, and determine the line integral of the electric intensity around the rectangle $drdx$. Let the electric intensity in the direction of x be denoted by P, and that in the direction of r by R; we then have

$$Rdr + \left(P + \frac{\partial P}{\partial r}dr\right)dx - \left(R + \frac{\partial R}{\partial x}dx\right)dr - Pdx = \frac{d}{dt}\cdot \mu T dr dx,$$

where μ denotes the permeability of the material composing the wire, and $\mu T dr dx$ therefore represents the total flux of induction through the area $drdx$. This reduces to the equation

$$\mu\frac{dT}{dt} = \frac{\partial P}{\partial r} - \frac{\partial R}{\partial x}.$$

If the specific conductivity of the wire is C, we have, as a consequence of Ohm's law,

$$u = CP,\ \rho = CR,$$

where ρ denotes the component current density in the direction of r. On substituting for P and R in the equation above, we finally obtain

$$\mu C\frac{dT}{dt} = \frac{\partial u}{\partial r} - \frac{\partial \rho}{\partial x}. \qquad (2)$$

3. In the interior of a conductor there can be no accumulation of electricity; or in other words, if we consider any small portion of a conductor, the amount of electricity flowing into this portion must be equal to the amount leaving it.

Consider any small volume bounded by the cylindrical surfaces whose radii are r and $r + dr$, and by two planes perpendicular to the axis of the wire. The quantity of electricity entering this volume during the time dt is $r\theta dr u dt + r\theta dx \rho dt$. That leaving the volume by the opposite faces is

$$r\theta dr\left(u + \frac{\partial u}{\partial x}dx\right)dt + (r + dr)\theta dx\left(\rho + \frac{\partial \rho}{\partial r}dr\right)dt.$$

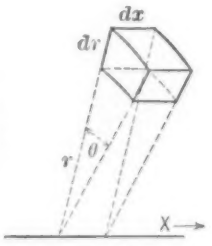


Fig. 3.

Since these quantities must be equal, we have

$$r\theta\frac{\partial u}{\partial x}drdxdt + \rho\theta drdxdt + r\theta\frac{\partial \rho}{dr}drdxdt = 0.$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\rho}{r} + \frac{\partial \rho}{\partial r} = 0. \tag{3}$$

We thus obtain three equations which must be satisfied by the various quantities involved, and these equations are seen to be nothing more than the analytical statement of the three physical laws quoted above. By elimination a condition is obtained which must be satisfied by the current density at all points. Elimination may be accomplished as follows:

Remembering that x and r are independent, equations (3) and (2) may be written

$$\frac{\partial}{\partial r}(r\rho) = -r\frac{\partial u}{\partial x}; \tag{3'}$$

$$-\frac{\partial}{\partial x}(r\rho) = -r\frac{\partial u}{\partial r} + \mu C\frac{d}{dt}(rT). \tag{2'}$$

If we differentiate (3') with reference to x, and (2') with reference to r, and add, we obtain

$$r\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + r\frac{\partial^2 u}{\partial x^2} - \mu C\frac{\partial}{\partial r}\frac{d(rT)}{dt} = 0. \tag{4}$$

By differentiating (1) with regard to t, we obtain

$$4\pi r\frac{du}{dt} = \frac{d}{dt}\frac{\partial(rT)}{\partial r}. \tag{1'}$$

T may therefore be eliminated between (1') and (4), giving finally

$$4\pi\mu C\frac{du}{dt} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2}. \tag{5}$$

The distribution of the current across the cross-section must therefore be such that the current density at all points satisfies the condition expressed in the above differential equation. As it is our intention to obtain a solution only for cases where the frequency of alternation is relatively small, viz. less than 100,000 per second, the equation might be still further simplified by the omission of the term in x; for it is clear that in circuits of moderate dimensions the current will have practically the same value at a given instant at all points along the wire. This is equivalent to saying that u is independent of x, and that the terms involving x may therefore be neglected. But by omitting this term we not merely restrict (apparently) the range of application of the resulting equations, but also lose sight of the manner in which the disturbance is propagated along the

wire. It therefore seems better to obtain a solution of equation (5) as it stands, and not to introduce approximations until later.

From physical considerations it is a matter of no great difficulty to determine the form which the solution of (5) must take in the present case. The experiments of Hertz have shown that if a periodic electromotive force is impressed at some point in a circuit, the disturbance will travel along the wire as a series of waves. The simplest possible expression for u would therefore be

$$u = M \cos 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right),$$

where τ is the period and λ the wave length. On account of the resistance of the wire, we should expect the waves to gradually diminish in intensity as their distance from the origin increases. This requires a factor of the form ϵ^{-ax} in the expression for u. Again, the amplitude and phase of the current wave will probably depend upon the distance of the point considered from the axis of the wire. To convince ourselves of this fact, we may think of the wire as a bundle of small conducting filaments, each one of these filaments forming a branch of the circuit in parallel with all the rest. The current will then divide among the different branches in a manner depending upon their mutual and self induction. From analogy with cases of linear circuits we may at once conclude that the different currents will differ both in amplitude and phase. Returning to a consideration of the actual wire, it thus appears that the current density at any point will vary harmonically with the time, but that the amplitude and phase will be functions of the distance of the point from the center. The current density may therefore be expressed in the following form:

$$u = \epsilon^{-at}\left[M \sin 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right) + N \cos 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right)\right], \qquad (6)$$

in which M and N are functions of r.

On substituting this expression for u in (5) and equating the coefficients of the sine and cosine terms separately to zero, two equations are obtained from which M and N can be determined. This method has been followed by Kelvin.

In this case, as in many other cases of the solution of differential equations, it is an advantage to proceed by a different method — a method which does not indicate so clearly the physics of the problem, but which makes the analytical work of solving the equation less tedious, and whose result is obtained in a form which lends itself more readily to numerical computation.

Remembering Euler's exponential expression for the sine and cosine, viz.

$$\epsilon^{-i\theta} = \cos\theta - i \sin\theta,$$

it is clear that (6) may be written in exponential form. If we write

$$\frac{2\pi}{\tau} = p, \quad \frac{2\pi}{\lambda} = m,$$

we may put u equal to the *real part* of

$$S\epsilon^{-\alpha x}\, \epsilon^{i(pt-mx)}, \tag{7}$$

where S is a function, in general complex, of r. The real part of S is evidently equal to the N of equation (6), while the imaginary part of S is equal to iM. If the expression above is substituted for u in equation (5), this becomes

$$\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} + (4\pi i\mu Cp + (im - \alpha)^2)\, S = 0. \tag{8}$$

The expression (7) is therefore a solution of (5) provided the function S satisfies the conditions indicated in equation (8). Since the coefficients of the original equation are all real, the real and imaginary parts of (7) must separately satisfy equation (6). We are therefore justified in using the formal solution (7) as long as there is any saving in manipulation by so doing, and may then take the real part alone as the solution of the actual physical problem.

To determine the values of α and m corresponding to any given frequency requires the dimensions and physical constants of the return circuit to be considered. A detailed discussion of several important cases has been given by J. J. Thomson.[1]

For the purposes of the present problem, however, it is not necessary to determine α and m; for both are small compared with the term $4\pi\mu Cip$. Since m involves the reciprocal of the wave length, it will be a small fraction unless the frequency is extremely great (10^8 or more per second). Experience teaches us that α, whose value depends upon the rate of damping of the waves, will also be small; p, on the other hand, will be large. The quantity $(im - \alpha)^2$ may then be neglected,[2] and (8) becomes

$$\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} + 4\pi i\mu CpS = 0. \tag{9}$$

The above is a form of Bessel's equation. If we write $4\pi\mu Cp = \phi^2$, the general solution is[3]

$$S = KJ_0(\phi r\sqrt{i}) + K'Y_0(\phi r\sqrt{i}), \tag{10}$$

[1] Recent Researches in Electricity and Magnetism, p. 262.

[2] Note that this leads to the same result as though the term in x had been omitted in equation (5).

[3] See Gray and Mathews' Treatise on Bessel Functions; or Byerly, Fourier's Series.

where K and K' are arbitrary constants. The function Y_0, however, becomes infinite when $r = 0$. This would mean an infinite current density at the center of the wire, which is evidently impossible. K' must therefore be equal to zero, and the solution reduces to

$$S = KJ_0(\phi r\sqrt{i}),$$

where J_0 is defined by the series:

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots.$$

The formal solution of equation (5) is therefore

$$u = KJ_0(\phi r\sqrt{i})\epsilon^{-ax-i(pt-mx)}. \qquad (11)$$

If we put

$$J_0(\phi r\sqrt{i}) = X - iY,$$

this becomes

$$u = K\epsilon^{-ax}(X - iY)\{\cos(pt - mx) - i\sin(pt - mx)\}.$$

As already explained, the solution of the actual physical problem is given by the *real* part of this expression. The proper expression for the current density at a distance r from the center of the wire is therefore

$$u = KX\epsilon^{-ax}\cos(pt - mx) - KY\epsilon^{-ax}\sin(pt - mx). \qquad (12)$$

Since we are concerned only with the distribution over the cross-section of the wire, any convenient value, as zero, may be assigned to x. If in addition we make the maximum value of u at the center of the wire equal to unity, the expression for the current density may be put into the following form:

$$u = X\cos pt - Y\sin pt,$$

or

$$u = u_0\cos(pt + \theta),$$

where

$$u_0 = \sqrt{X^2 + Y^2},$$

$$\tan\theta = \frac{Y}{X},$$

and

$$X - iY = J_0(\phi r\sqrt{i}) = J_0(\sqrt{i}\sqrt{4\pi\mu Cpr^2}).$$

It is thus seen that the maximum value of the current density, and also the phase of the current at any distance r from the axis, depends upon X and Y only; while X and Y in turn depend only upon the value of $4\pi\mu Cpr^2$. In recording the results of numerical computation it is therefore convenient to tabulate the values of u_0 and θ corresponding to a series of values of the expression $4\pi\mu Cpr^2$. By giving proper values to the constants μ, C, p, and r, such a table may be made to give numerical results for any special case within its range. The values of $J_0(q\sqrt{i})$ up to $q = 6$

are given by Gray and Mathews,[1] while a table extending, with greater intervals, to $q = 20$, has been computed by Mr. Magnus Maclean.[2] By the aid of the values for X and Y given by these writers, I have computed u_0 and θ for different values of q, where $q = 4\pi\mu Cpr^2 = 8\pi^2\mu Cnr^2$, n being the frequency. The results are given in the accompanying table.

TABLE I.

q	X	Y	u_0	θ (in degrees).	$\frac{R_n}{R_s}$
0.	1.000	0.000	1.000	0.	1.000
0.5	0.999	0.062	1.001	3.5	1.000
1.	0.984	0.250	1.015	14.3	1.000
1.5	0.921	0.558	1.077	31.2	1.026
2.	0.752	0.972	1.229	52.3	1.080
2.5	0.400	1.457	1.511	74.6	1.175
3.	− 0.221	1.938	1.96	96.5	1.318
3.5	− 1.194	2.283	2.58	117.6	1.492
4.	− 2.563	2 293	3.44	138.1	1.678
4.5	− 4.299	1.686	4.62	158.6	1.863
5.	− 6.230	+ 0.116	6.23	178.9	2.043
5.5	− 7.973	− 2.790	8.45	199.3	2.219
6.	− 8.858	− 7.335	11.5	219.6	2.394
8.	+ 20.97	− 35.02	40.8	300.6	3.096
10.	− 138.8	+ 56.37	150.	382.1	3.794
15.	− 2970.	− 2952.	4190.	584.8	5.573
20.	+11500.	+47580.	48900.	796.4	7.325

EXPLANATION OF TABLE.

$q = \sqrt{4\pi\mu Cpr^2} = \sqrt{8\pi^2\mu Cnr^2}$, where μ is the permeability, C the specific conductivity, and r the distance of the point considered from the center of the wire. n represents the frequency. Then u_0 is the value of the maximum current density, the current density at the center being taken equal to unity. θ is the phase difference between the current at the distance r from the center and the current along the axis of the wire.

$\frac{R_n}{R_s}$ is the ratio between the resistance offered to alternating currents of frequency n, and the resistance offered by the same wire to steady currents.

$$X - iY = J_0(q\sqrt{i}).$$

As an illustration of the use of this table, consider the following special case: A copper wire 1 cm. in diameter carries an alternating current

[1] Treatise on Bessel Functions, Table IV.

[2] See Kelvin, Mathematical and Physical Papers, Vol. III., p. 493.

whose frequency is 2000 per sec. How does the current density at the surface compare with that at the center?

We have in this case,

$$\mu = 1, \quad C = \frac{1}{1600}, \quad n = 2000, \quad r = 0.5,$$

$$q = \sqrt{8\pi^2 \mu C n r^2} = 4.97.$$

The value of u_0 corresponding to $q = 4.97$ is very nearly 6.1. The maximum current density at the surface of the wire is thus six times as great as the maximum value at the center. The phase difference θ is in this case nearly 178°. It thus appears that the current at the center of the wire lags behind the current at the surface by almost 180°; so that when the surface current is at its maximum in the positive direction the current at the center of the wire has just reached its maximum in the negative direction.

By giving different values, ranging from 0 to 0.5, to r, the value of u_0 may be found for all points in the wire. The dotted curve in Fig. 4 has been plotted from results obtained in this way, distances from the center of the wire being used as abscissas, and the corresponding values of u_0 as ordinates. To avoid interpolation, n has been taken equal to 2020, thus making $q = 5$ at the surface.

While this curve indicates fairly well the magnitude of the throttling effect, it fails to give an entirely satisfactory picture of the actual condition of affairs in the wire. On account of the varying phase angle θ, the current density does not reach its greatest value at the same instant at all points of the cross-section, so that the dotted curve in Fig. 4 does not show the actual distribution of current at any definite time. To find the distribution at any given instant, we must assign the proper value to u_0 and θ in the expression

$$u = u_0 \cos\left(\frac{2\pi t}{\tau} + \theta\right),$$

and thus determine u for different values of r, but for the *same* value of t. If these results are plotted, we obtain what might be termed an instantaneous view of the current distribution. Three such curves are shown in Fig. 4. The first shows the condition of affairs at the instant that the surface current has reached its positive maximum. At a distance of 2.8 mm. from the center of the wire the current at this instant is zero, while at all points still nearer the axis the current flows in the opposite direction. It is clear that so far as outside magnetic effects are concerned, the current in the interior of the wire tends to neutralize the effect of the surface current. In this case the presence of the interior portion of the conductor is not merely of small utility; it is actually a disadvantage. The second

curve in Fig. 4 gives the distribution at the instant that u at the surface is passing through zero in the negative direction. In Curve III the surface current is approaching its negative maximum.

For a frequency of 2000 the values of θ are too small to bring out clearly the manner in which the current is propagated into the wire. By taking $n = 8080$, which makes $q = 10$ at the surface of a copper wire 1 cm. in diameter, we obtain a case where all the phenomena of throttling are much more marked. The maximum current density at the surface is now 150 times as great as that at the axis, while the phase difference between the two currents is 382°. The variation of u_0 with r in this case is shown

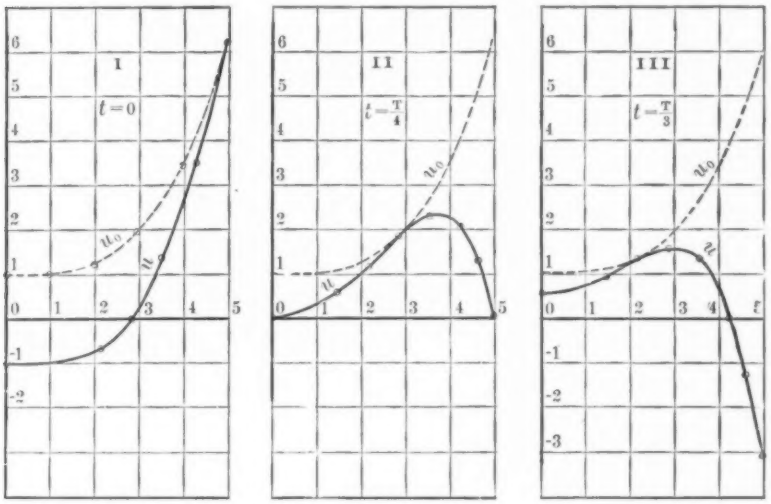


Fig. 4.

in Fig. 5, IV. The dotted curve in the same figure gives the value of θ at different distances from the axis. It will be observed that the latter curve, with the exception of the region near the center, is practically straight. We conclude from this that the current density at points not too near the axis lags behind the surface current by an angle which is proportional to the depth below the surface. The disturbance which constitutes the current thus travels inward from the surface of the wire in the form of a wave, whose velocity of propagation remains practically constant until the wave has almost reached the center. The amplitude, on the other hand, diminishes very rapidly as the depth below the surface increases. The curves plotted in Fig. 5 have been drawn to illustrate this point. In Curve I we have the distribution at the instant when the surface current is passing through zero in the positive direction. Curve II shows the condition

Distribution of an Alternating Current in a Copper Wire.
Diameter of wire = 1 cm.; Frequency = 8080 periods per sec.

In each curve the abscissa gives the distance from the center of the wire, while the ordinate gives the corresponding value of the current density. Seven phases are shown, each curve showing the distribution at a time $\frac{1}{12}T$ later than the preceding curve.

Fig. 5.

of affairs $\frac{1}{12}$ period later; Curve III gives the distribution at the end of another $\frac{1}{12}$ period interval; and so on. Figure 5 thus shows seven successive phases of the disturbance in the wire, and enables the phenomenon to be followed through half an oscillation. It may be added that the next half oscillation will differ from that shown only in the fact that the sign of u is reversed.

Only a brief inspection of Fig. 5 is necessary in order to appreciate the manner in which the current is propagated into the wire. The wave of current, starting in Curve I near the surface of the wire, may be followed through its successive stages as it proceeds, with rapidly diminishing amplitude, toward the axis. These diagrams also enable the velocity of propagation to be determined. If we compare Curves I and VII, we see that the zero point of the current wave has moved from $r = 5$ mm. to $r = 2.82$ mm. The wave has therefore traveled a distance of 2.18 mm. in $\frac{1}{1600}$ sec. This corresponds to a velocity of 35 meters per second.

In general, the velocity with which the disturbance is propagated inward will depend upon the frequency of the current as well as upon the material of the wire. The velocity will not be strictly constant unless the diameter of the wire is infinite; but for points in the wire for which q exceeds 2 it will be so nearly constant that its variation is negligible. A general expression for the velocity is

$$V = \sqrt{\frac{n}{\mu C}}.$$

This expression is obtained on the assumption that $r = \infty$; *i.e.* for a plane conducting sheet.[1] That it holds with a high degree of approximation even for rather small wires may be verified by the use of the table given above.

There results from this concentration of current at the surface a real change in the resistance of the wire — real in the sense that it corresponds to an increased development of heat, and not merely to a change in the impedance of the circuit. The central portion of the wire no longer contributes its share to the conductivity, so that the resistance is practically that of a thin cylindrical tube. In the last column of the table on p. 55, I have tabulated the values given by Kelvin[2] for the effective resistance offered by cylindrical wires to alternating currents. The resistance R_n for an alternating current of frequency n is here expressed in terms of the resistance R_0, which the same wire offers to steady currents. In Fig. 6 these results are shown graphically for the case of a copper wire 1 cm. in diameter.

[1] See Kelvin, "Anti-effective Copper," London Electrician, Vol. 25, p. 512, 1890.

[2] Mathematical and Physical Papers, Vol. III., p. 492.

On account of the high permeability of iron the various effects of throttling are shown by wires of that material even when the periodicity is quite low. If we assume a permeability of 1500, which is a low value for soft iron, the curves of Fig. 5 would apply to an iron wire 1 cm. in diameter when the frequency is only 32 per second. In the case of the magnetic metals, however, the value of μ cannot be treated as constant. The permeability in such cases is a function of the current, and, if hysteresis is

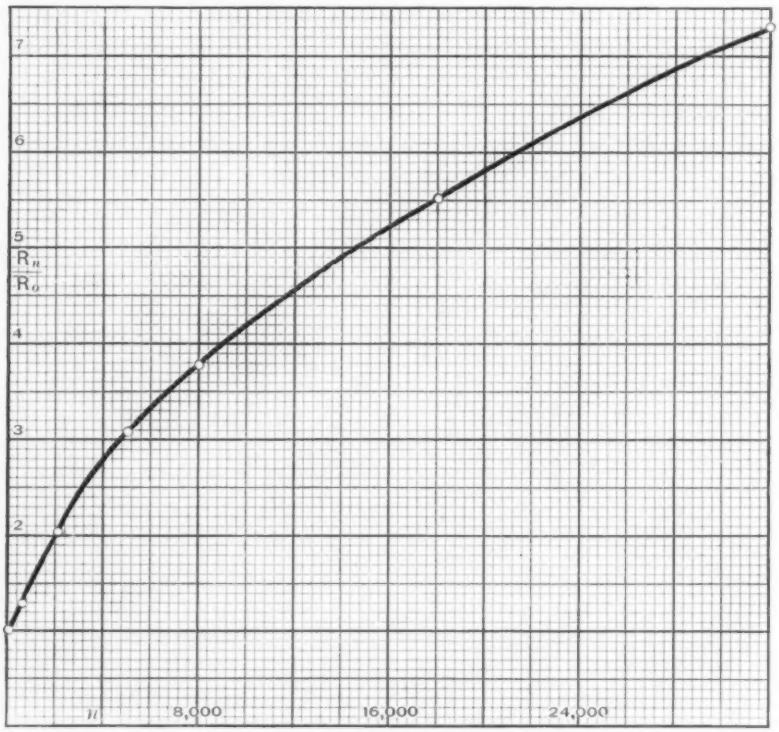


Fig. 6.

appreciable, of the rate of change of the current also. The results derived above can therefore not be expected to show an exact agreement with experiment in the case of iron wires, although they doubtless give a rough approximation to the truth. The subject of the distribution of alternating currents in wires of the magnetic metals is in fact one which deserves further experimental study. It can hardly be doubted that such an investigation would be a valuable contribution to our knowledge of magnetism.

NEW BOOKS.

What is Electricity? By JOHN TROWBRIDGE, S.D. (The International Scientific Series, No. 75.) New York, D. Appleton & Company, 1896.

It is a little singular that this well-known series should have reached its seventy-fifth number without including a volume on any electrical subject. No less than five have been devoted to various branches of optics, and several to other departments of physics. Professor Trowbridge's book, though the first on this topic, can hardly be considered as an introductory treatise. It reviews the whole field of electrical phenomena — and many other things besides; but it aims to present them in such a way "that the reader can perceive the physicist's reasons for supposing that all space is filled with a medium which transmits electromagnetic waves to us from the sun."

The treatise indeed, as the author states in another place (p. 4), is intended to be "a popular presentation of what I regard as the real subject of physics, — the transformations of energy, — and of the greatest generalization in that subject (Maxwell's Theory of Electromagnetic Origin of Light and Heat)."

While the theme is highly attractive, it is not an easy one for popular treatment, because of the large amount of preliminary information which is necessary to any really definite conception of electrical oscillations.

The essence of the book lies in the later portions. In them, from the chapter on alternating currents, are discussed successively Leyden-jar discharges, lightning, step-up transformers. Then the author, drawing nearer the heart of his theme, describes the high-frequency experiments of Tesla and Elihu Thomson, passes to the subject of electrical waves, and the work of Hertz, Lenard, and Röntgen, and, after a short discussion of the electromagnetic theory of light, closes with a résumé of electrical theories in general.

These chapters, to him with sufficient knowledge to understand them, open the way into a new and fascinating field of physical research. The experimental illustrations are striking and ingenious. The phenomena described have not hitherto been grouped together in one treatise, and many of them have been almost inaccessible to the general reader. The

very richness of the subject, however, has brought its own difficulties. The amount of ground gone over is so great that the explanations are often hasty and inadequate, sometimes even incorrect. In the chapter on wave motion — to give a single instance — is brought together a great number of curious and interesting experimental results, so briefly described as to be practically unintelligible to a reader without a previous knowledge of physics. Here within the compass of two dozen duodecimo pages are discussed Wiener's photographs of standing waves in a film, Lippmann's color photographs, Boys' photographs of flying bullets, Töpler's method of rendering sound waves visible, Langley's bolometric researches, and many others. Michelson's interference apparatus is described in six lines, and the description is as erroneous as it is short.

The first eight or ten chapters treat of the more ordinary and well-known electrical manifestations, and must be regarded as introductory to the subject proper. The extremely elementary nature of some of them — such as those on the galvanometer and dynamo — shows that no previous knowledge of electricity is assumed on the part of the reader. To these early chapters he must look for whatever principles and definitions he may need to comprehend properly the portions which follow, yet he will look in vain for definite information upon many important topics, which are referred to elsewhere, without explanation.

More to be regretted than even such omissions, are the half-truths, misleading statements, and errors, which are numerous.

The "absolute system" of measurements is explained (p. 11) as "absolute in the sense that everything is referred to acceleration of a body falling to the earth at a definite place through a certain space in a definite time." This is not easily reconciled with the ordinary conception of an absolute system, which is explained by Andrew Gray as "a system of fundamental units in no way affected by locality or other conditions of experimenting." The definitions of electrical units (p. 62) are unintelligible as they stand, though of course the meaning is clear enough to one who already knows it.

It is at least misleading to say that a "compass needle instantly points to the wire carrying the current" (p. 47), or that magnetic forces are proportional to the products of the attracting "masses" (p. 107).

It is said of polarizing bodies — a Nicol prism is instanced as an example — that "we can think of such substances as similar in internal construction to a Venetian blind with its horizontal slats" (p. 80). Of course we *can* think so of it if we like, but it is hardly fair to leave upon the trusting reader's mind the impression that the Venetian blind is anything more than a crude illustration. Even less excusable is the explanation of diffraction through a slit, that the bright and dark lines extending each

side of the slit "are due to the waves of light which emanate from the illuminated edges of the slit, and which interfere" (p. 223).

Statements of a similar character are to be found in the description of the diffraction grating (p. 230) and of the Rowland concave grating (p. 232). Many others might be cited. The beginner who takes this book for a guide will have much to unlearn, should he ever wish to carry his reading further.

The question which forms the title of the book is nowhere directly answered, but in several places statements are made which seem to throw light on the author's opinion.

On p. 49 it is said that "the only difference between light, heat, and electricity is in the length of waves," and on p. 105, "Our principal source of electricity to-day is the steam-engine; yet it can be maintained that electricity is back of the steam-engine. It is in the coal" — and again (p. 128), "Nature sends us, so to speak, our electricity by means of to-and-fro motions from the sun." In several places also the "velocity of electricity" is said to be the same as the velocity of light.

Judging from these statements, Professor Trowbridge holds electricity to be a form of energy, which, in the present state of electrical theory, is a hard saying.

FRANK P. WHITMAN.

James Clerk Maxwell and Modern Physics. By R. T. GLAZEBROOK, F.R.S. (The Century Science Series.) New York, Macmillan & Company, 1896.

The life and scientific work of James Clerk Maxwell has already been so admirably presented that a new biography might seem superfluous. There is little indeed to be found in the first portion of the present volume, which is biographical in its character, that has not appeared either in the work of Campbell and Garnett,[1] or the preface to Maxwell's Collected Works,[2] by Niven.

Professor Glazebrook, than whom no one is better entitled to speak of Maxwell, has, however, gathered together, within the compass of less than a hundred pages, materials which give a very vivid and altogether pleasing picture of Maxwell's short but extraordinary career; and his sketch, because of its brevity, will doubtless be read by many who would never have opened the larger work of Campbell and Garnett.

[1] *The Life of James Clerk Maxwell*, by Lewis Campbell and William Garnett, London, Macmillan & Co., 1882.

[2] *Scientific Papers of James Clerk Maxwell*, edited by W. D. Niven, Cambridge, 1890.

The latter half of the volume is devoted to a *résumé* of Maxwell's work. To give any adequate expression in popular language to the results of profound investigations, such as those upon which the fame of Maxwell rests, is an exceedingly difficult task. This task has been admirably done. Those who are acquainted with the researches in their entirety will read this lucid presentation of them with great pleasure; while every intelligent reader will be able to gain a definite conception of the general scope of Maxwell's contributions to science.

E. L. N.

Humphry Davy, Poet and Philosopher. By T. E. THORPE, LL.D., F.R.S. (The Century Science Series.) New York, Macmillan & Company, 1896.

Chemistry has made such strides since the beginning of the century that an intelligible account of the labors of Sir Humphry Davy was much needed. Professor Thorpe's little book gives a more accurate account of the man, and of his work, than can be derived from the reading of the biographies written by Paris and by John Davy, or from the sketch contained in Brougham's *Lives of Men of Letters and Science.*

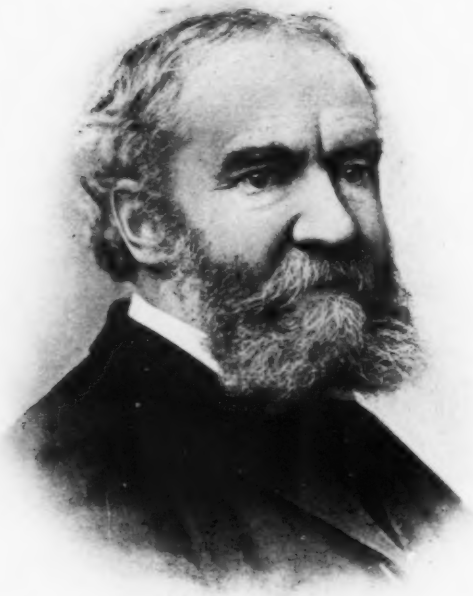
It is easier for the writer to-day to estimate the value of Davy's labors, and to obtain an impartial view of his character and motives, than it was for those who were his contemporaries. As to the place which should be assigned to his work in the history of the development of physics and chemistry, it was indeed impossible to speak impartially, until the entire era to which his labors belonged had become historical. In this still somewhat delicate matter Professor Thorpe has been very skillful. His account of Davy, the man, is appreciative and discriminating, and at the same time quite free from any elements of hero worship. In his description of Davy's discoveries he has made many points, otherwise obscure, intelligible to those to whom the terminology of that great transition period in chemistry, the beginning of the century, is unknown.

E. L. N.

NOTICE: CHANGE FROM ANNUAL TO SEMI-ANNUAL VOLUMES.

Beginning with the present number, two volumes of the PHYSICAL REVIEW will be published annually. These volumes will begin in July and January respectively, and will each contain at least five numbers.

The past volumes of the PHYSICAL REVIEW (I.–IV.) are annual volumes, each containing six bi-monthly numbers, beginning with the July-August number.



Alfred M. Mayer

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